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THE JOULE-THOMPSON EFFECT IN SUPERHEATED STEAM: I. EXPERIMENTAL STUDY OF HEAT-LEAKAGE.

BY H. M. TRUEBLOOD.

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(Continued from page 3 of cover.)

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By H. M. Trueblood.

Received, January 14, 1917.

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I. INTRODUCTION.

Some time ago, the writer undertook the experimental determination of the Joule-Thomson effect and of the product of this by the specific heat at constant pressure, in superheated steam, as functions of pressure and temperature. The theoretical and practical importance of such measurements as these has been fully set forth in a recent paper by Davis.[1] In the experimental work, it became evident with the first results obtained that the principal problem was the reduction of heat leak to such a point that its effect might be eliminated without an excessive number of observations under varied conditions of pressure drop and flow at a single point of the (p, T) plane. The experimental study of this problem developed chiefly in the direction of making measurements of μ, the Joule-Thomson coefficient, with plugs of different types, under a considerable variety of circumstances in respect of the arrangement of the containing case, lagging, etc., the determination of the product μC_p being temporarily abandoned. These experiments have involved altogether nearly one hundred separate measurements of μ with three distinct types of radial flow plug case, one of which was especially designed for the purpose of studying the effect of variations in lagging, and with one type of case for what may be called, for the sake of convenient distinction, an *axial* flow plug. Of these measurements of μ, all but fourteen were made at practically the same pressure and temperature.

The chief intent of the present paper is the presentation and discussion of the results of these measurements of μ at a single point of the plane, as a study of the question of heat leak in throttling calorimetry, and particularly as a study of the behavior of the radial flow plug as applied to throttling calorimetry; for while a considerable proportion of the runs were made with axial flow plugs, the general type of plug and containing case used in all of these was of an early design, in which the heat leak was unnecessarily large, so that the interest these axial flow runs have for the purposes of this paper lies chiefly in the fact that they illustrate rather strikingly the heat-leak possibilities in throttling apparatus not designed primarily to avoid heat-leak, rather than in any basis they afford for a fair comparison of the relative merits of correctly designed axial flow and radial flow plugs.

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1 Davis, Phys. Rev. (2) **5**, 359 (1915).

While the results obtained with the latest type of radial flow apparatus used justify the conclusion that the heat leak problem has been satisfactorily solved, there is little of value, in the results here presented, to those who are primarily interested in the thermodynamics of superheated steam, since the work covers the determination of μ at only one point. The experimental work at other points is now in progress, and it is expected that its results will be set forth in a future paper. This further prosecution of the work has been made possible by the generous loan of most of the necessary apparatus to the writer by the Director of the Jefferson Physical Laboratory of Harvard University. All of the results presented and discussed in this paper were obtained in the Jefferson Laboratory.

II. THE THROTTLING APPARATUS.

In all the work described and discussed in this paper, the throttling apparatus, whether of the radial or axial flow type, was immersed in an oil bath, the temperature of which was automatically maintained constant. This oil bath also contained a device, hereafter called the secondary superheater, consisting of a coil of pipe or of pipes, through which the steam was passed before reaching the high side of the plug. With this arrangement, very common in throttling calorimetry, the temperature of the fluid on the high side of the plug is presumably that of the bath, at least if the velocity of flow of the fluid when in the secondary superheater is the same as it is at the high side thermometer bulb. For fluids with positive Joule-Thomson effects, the heat leakage is therefore from the bath inward to the fluid, if the fluid is merely throttled without other addition of heat.

1. Axial Flow Apparatus.

The general type of axial flow porous plug (a term here used in contradistinction to the term 'radial flow plug,' which is the other fundamental type involved in these experiments) is illustrated in Fig. 1. The steam enters at one of the orifices near the top of the cap C, passes the high side thermometer (not shown, but screwing into the bushing M), thence through the numerous fine holes shown in section in the soapstone block S, through the cross channel in the

soapstone block S', through another set of holes in block S similar to those shown, past the low side thermometer bulb, and out through an orifice (not shown) in cap C'. The head of the low side thermometer may be seen projecting from the cap C'. The two sets of holes in the block S are threaded with a continuous piece of wire, used as a

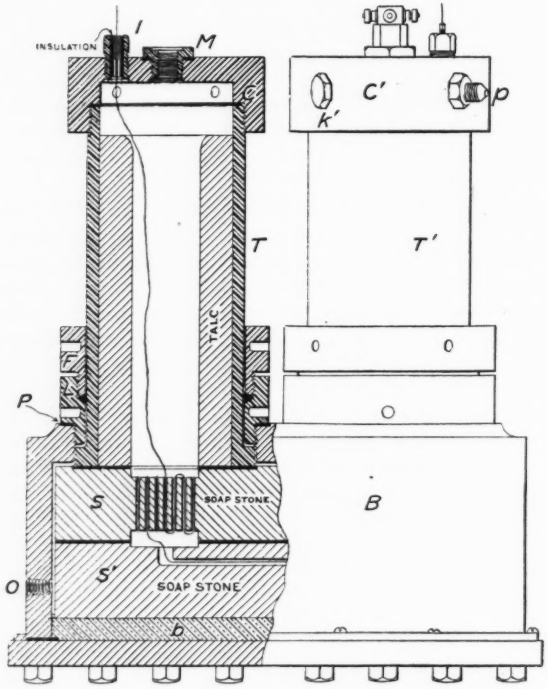

FIGURE 1. Axial flow plug and plug case.

heating coil when it is desired to conduct the experiment isothermally for the measurement of μC_p. This wire in certain plugs was of solid manganin, in size approximately No. 18 B. and S. gauge; in others, it was of stranded invar, ten strands, each about 0.01 inch in diameter, being used. Connections for the differential pressure gauge are

taken from the bushing p and from a similar bushing on the high side.

All axial flow plugs were of this general form and were contained in the plug case shown. The plugs used differed in the size of the holes drilled in S, in the dimensions and position of the cross channel in S', in the character of the lagging in the vertical tubes T and T' (which was of talc in some plugs and of 'poplox' in others) and in the dimensions of the chamber between S and S'. In certain plugs this chamber, somewhat enlarged for the purpose, was filled with copper baffle-plates, the object being to secure a more intimate mixing of the steam in its passage through the plug. In other plugs, these baffle plates were replaced by a number of layers of fine copper gauze. In certain plugs, additional baffles consisting of brass plates with intervening layers of copper gauze were located in the vertical tubes T and T' just above the holes in the block S. Various combinations of these baffling devices were tried, chiefly to see whether any noticeable effect in the observed values of μC_p with varying intimacy of steam-mixing could be detected. The results of these experiments were in general negative, no consistent variations in the results on μC_p being traceable to changes in the baffling.

In the plugs in which 'poplox' was used for lagging in the vertical tubes, the poplox, which is a very light cellular material, was contained between two thin concentric brass tubes (corresponding to the inner and outer surfaces of the annular cylinder of talc shown in the tube T of Fig. 1). The inner of these tubes was used to afford an electrical connection between the heating coil and the insulating plug I, the wire lead shown in the figure being dispensed with. This was done because it was suspected that some part of certain rather large discrepancies in the measured values of μC_p might be due to an accidental proximity of the lead wire, heated by the passage of the electric current, to the bulb of the thermometer on one side or the other of the plug.

The plug-case was swung in a cradle which also supported the secondary superheater, consisting in this case of a helical coil of about 40 feet of $\frac{3}{8}$ inch copper pipe (actual inside diameter = 0.49 inch) of about the height of the plug case and arranged so that it enclosed the plug case. This coil of pipe was connected at one end to the high side of the plug, at the other to a primary superheater (see Figs. 5 & 6) from which the supply of steam was drawn. The cradle, with the plug case, coil of pipe and two motor-driven stirrers, was suspended by means of a block-and-falls in the oil bath which has already been mentioned.

The most obvious defect in the type of plug shown in Fig. 1 is, of course, the opportunity for heat leakage afforded by the wide separation of the two thermometer bulbs. Nearly half of all the throttling has taken place before the steam enters the cross-channel, in which the temperature of the steam is consequently considerably less than that of the oil. This defect was not unnoticed in designing the apparatus, but its importance was much underrated. It was expected that the fairly good thermal insulation of the steam while in the cross-channel would prevent heat leakage of inconvenient magnitude (the soapstone blocks of Fig. 1 are about two inches thick and over four inches wide), and the first practicable apparatus used was built in this form, chiefly because it could be easily handled and because the design permits an easy and rapid interchange of the (resistance) thermometers for the purpose of eliminating their normal difference of resistance. Soapstone, however, is not a particularly good heat insulator, and it was found impossible to obtain reliable values of the Joule-Thomson coefficient with plugs of this type without an excessive amount of variation in the conditions of flow and pressure drop at each pressure and temperature of measurement.

2. Radial Flow Apparatus.

a. General remarks on the radial flow plug.

The theory of the radial flow porous plug has been discussed in a paper by Burnett and Roebuck.[2] One of these authors has since applied it to the experimental determination of the mechanical equivalent of heat.[3] A radial flow plug was also used by Regnault in some porous plug experiments on air. He regarded his results as very unsatisfactory and the time spent in securing them as wasted. So far as is known to the writer, no other experimental results obtained with the radial flow plug have been published. Briefly, the radial flow plug consists of a thin cup of porous material of low thermal conductivity — porcelain, for example — and of a length some 6 or 8 times its diameter (see Fig. 2). The fluid under experiment is made to flow through the cup from the outside to the inside. The low-side thermometer bulb is located near the bottom of the cup on the inside.

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2 E. S. Burnett and J. R. Roebuck, Phys. Rev. (1) **30**, 529 (1910).
3 J. R. Roebuck, Phys. Rev. (2), **2**, 79 (1913).

The high side thermometer bulb may be placed at some convenient point in the path of the fluid ahead of the cup, preferably as close to the cup as possible. The whole is immersed in a bath, the temperature of which is that of the fluid at the high side thermometer. If the Joule-Thomson effect in the fluid is positive, the conductive flow of leaking heat (aside from conduction down the thin, long and poorly-conducting walls of the cup — a relatively small item) is in the direction of the flow of the fluid, and ultimately consists in the transfer of heat from a thin layer of the fluid immediately adjacent to the plug wall on the high pressure side to a similar layer on the low pressure side. This transfer of heat obviously has no effect on the temperature drop between thermometers after the steady state has been established, provided the thin layer of fluid on the outside of the plug is prevented from receiving heat through the metallic walls of the plug case from the bath. Aside from radiation, which is doubtless

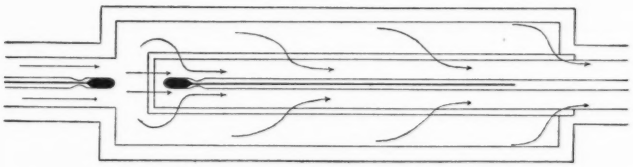

FIGURE 2. Diagram of radial flow plug.

slight, heat may be gained, from the bath, by the fluid at any instant adjacent to the high side of the plug, by conduction, provided the temperature depression on the high side due to conduction through the plug gives rise to an appreciable temperature gradient at the walls of the plug case; that is, provided the "thin layer" of fluid mentioned above is sufficiently thick. If this is the case, any mass of fluid will, of course, also acquire heat before reaching the high side of the plug, while it is moving through regions in which the temperature is not that of the bath. Theoretically, at least in the ideal case of an infinitely thin plug of material having a finite thermal conductivity, the thickness of the "thin layer" is zero*, in the steady state, and while it is probably small, at least in the vicinity of the closed end in such

* The analysis on which this statement is based ignores changes in volume and specific heat due to the throttling, and also changes in temperature caused by the increased average velocity of the fluid while passing through the plug.

plugs as have actually been used, the experiments to be discussed later show that it is essential to shield the fluid from the effects of this conductive action by interposing lagging between the plug and the walls of the containing case (see Fig. 4).

An important effect which is analogous in its origin to the effect due to heat conduction through the body of the plug arises, if certain precautions are not taken, from the rather peculiar geometry of the radial flow plug. At its open end, the plug is necessarily brought, for purposes of mechanical support, into good thermal contact with the metal walls of the case. As a result, fluid which has passed the low side thermometer is provided with a thermally short path of communication with fluid which has not yet passed through the plug. This results in a depression of the temperature of the latter, which, joining the low side fluid at a temperature therefore lower than then exists there, contributes to a still further depression of the temperature of the fluid which has not yet passed. This cumulative action, similar in principle to that taking place in certain types of liquid air machines, is limited by the ability of the bath to supply heat to the low side fluid. The reason that a similar action does not take place by conduction through the plug walls is that, at least near the closed end of the plug, the direction of flow is parallel to the temperature gradient in the plug for a considerable distance on both sides of the plug; undoubtedly the effect does exist in some degree near the open end of the plug, where the stream lines even a very short distance from the plug walls are approximately axial.

The effect of the action just described on the observed temperature drop is indirect, since the fluid directly affected does not pass the low side thermometer. But the temperature depression produced in the neighborhood of the open end produces indirectly, by conduction through the walls of the plug, the walls of the case, the inside lagging and, doubtless, to some extent through the fluid itself, a depression of the temperature of fluid which does pass the low-side thermometer; that is, there is an outward leak of heat from fluid between thermometers to fluid which has passed the low side thermometer, with the result that the observed temperature drop is too high.

This effect, of the existence of which experimental evidence is given later (see IV, 2, *d*) will be called the 'regeneration effect' hereafter. The remedy for it is to provide sufficient lagging between the metallic support of the plug and the high side fluid. This lagging will be called 'internal end-lagging' or simply 'end-lagging' whenever it is necessary to refer to it in what follows.

b. The V-type of radial flow plug case.

The apparatus illustrated in Fig. 3 will be referred to as the V-type of plug case, for the sake of brevity. The design, which is due to H. N. Davis, under whose direction the apparatus was also constructed, was adopted with the two-fold purpose of retaining the convenience of approximately vertical thermometers and of doing away with the

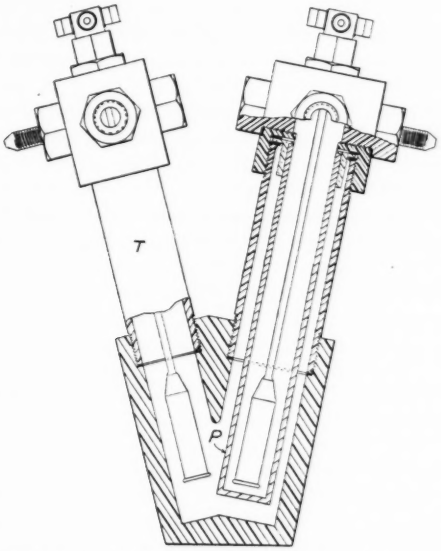

FIGURE 3. Radial flow plug and case, V-type.

lengthy cross channel of Fig. 1. It was also possible to employ the oil tank, stirrers, cradle, secondary superheater, etc., which had been used with the apparatus of Fig. 1.

The plug *P*, in Fig. 3, is of alundum, and was obtained from the Norton Alundum Co., of Worcester, Massachusetts. It was secured, as shown, to a brass collar by the device of plating from a solution of copper sulphate onto the inside of the collar, through the alundum, from a copper electrode placed just inside the open end of the cup. The copper fills up the pores of the alundum and a mechanically rigid

union is thus obtained. This method of attachment is also due to Prof. Davis and is very satisfactory. It was deemed advisable to make the joint steam-tight by the use of bakelite varnish, bakelized under pressure according to standard methods of treating this material. Bakelite varnish has also been depended upon for mechanical union in some plugs, the copper plating being dispensed with; but the results are not so satisfactory as when the plating method is used.

No internal lagging whatever was used with this type of plug case; but the case was provided with a galvanized sheet iron container (not shown) resembling a small coal scuttle. The plug case being placed inside the container, the space remaining could be filled with poplox, giving an average thickness of insulation of perhaps two inches between the plug case and the oil. The container was equipped with an oil tight top, so that it, with the plug and poplox inside, could be completely immersed in the oil bath. Experiments were conducted, (i) without the container; (ii) with the container, but no poplox, the cover being omitted and the container immersed until its upper edge was flush with the surface of the oil; (iii) with the container filled with poplox, cover omitted, immersion as in (ii); (iv) with the container filled with poplox, covered and completely immersed.

c. The U-type of radial flow plug case.

This name is used merely for convenience. The plug case was simply the axial flow case of Fig. 1, with certain minor changes necessary for adaptation to radial flow work. The plug was located in the low side tube, T' of Fig. 1, and was supported by means of a special flange which made possible complete internal lagging, of a thickness roughly $\frac{1}{2}$ inch at the open end of the plug and rather less than $\frac{3}{4}$ inch elsewhere. The end-lagging was asbestos wood (a commercial product consisting of asbestos fibres with a binding of portland cement); the other lagging, separating the plug from the steel wall of the tube T', was of talc, as in Fig. 1. It may be mentioned that asbestos wood is a much more efficient lagging material than either talc or soapstone.

The block S of Fig. 1 was replaced by a block having a single large hole at the place at which the heating coil is shown in Fig. 1 and a similar hole at the corresponding place on the low side. The block corresponding to S' had a half-inch channel, with baffling and mixing chambers at each end.

Only a few runs were made with this type of plug case, which was

used because it was the simplest form of apparatus immediately available at the time by which internal lagging could be provided. It was found that although the regeneration effect noticed with the V-type of case was absent, or practically so, a considerable error due to heat leak in the cross-channel, and through the talc lining of the tube T', persisted. This seems to have been due mainly to the depression of temperature experienced by the steam in passing through the baffling chambers. The most important bearing which the runs made with this apparatus have is an incidental one concerned with the question of the dryness of the steam; this is discussed later (see IV, 2, *g*).

d. Straight-away type of plug case.

The final form of plug case adopted, and the one by means of which the various heat-leak difficulties are believed to have been solved, is illustrated in Fig. 4. It will be seen that the annular lagging (that concentric with the plug) is quite heavy; it is nearly two inches thick. The slight axial taper given to its inner surface is intended to prevent variation in the velocity of axial flow along the outside of the plug. The apparatus is designed throughout to preserve a nearly constant velocity of flow, equal to that in the secondary superheater. The end-lagging (that at the open end of the plug) is also thicker than in previous apparatus, being somewhat over one inch in thickness. All lagging is of asbestos wood.

The two thermometer bulbs are placed almost as close together as it is possible to get them. Their distance of separation may be varied, if desired, by using longer bushings at *B*, *B*.

The plug case is designed so that it may also be used for an axial flow plug, if desired. The change is effected by replacing the radial flow plug and its lagging with a suitably lagged axial flow plug. The low side cap is removed and replaced with a coupling, into which is screwed a cylindrical chamber of about the same dimensions as that which, in Fig. 4, is shown containing the radial flow plug and its lagging. The low side cap is then screwed onto the other end of this new chamber, which contains the heavily-lagged low-side thermometer. No runs have yet been made with this axial flow set-up.

III. OTHER APPARATUS.

A general view of the essential parts of the whole apparatus is given in Fig. 5 (plan, with end elevation showing manometer connections)

and Fig. 6 (front elevation). The oil bath and its contents (secondary superheater, stirrers, etc.), shown in these figures are as used with the straight-away type of plug described under II, 2, *d* above. A complete reconstruction of all of this section of the apparatus was necessitated by the adoption of this type of plug. The remainder of the apparatus, aside from a few minor changes, was unaltered by the change from the vertical to the horizontal type of plug.

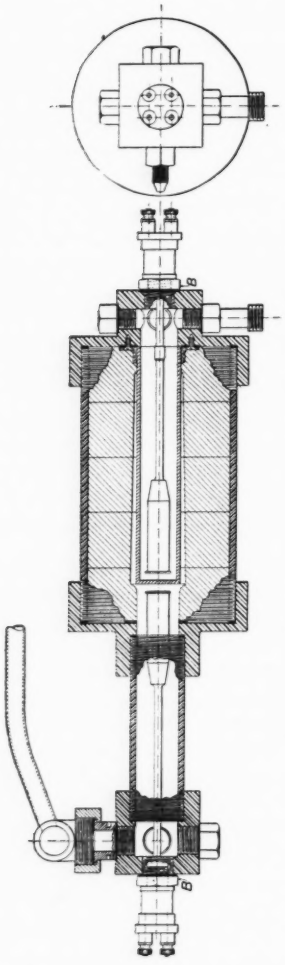

FIGURE 4. Straight-away Type of Radial Flow Plug Case.

1. OIL BATH AND SECONDARY SUPERHEATER.

Only the bath and superheater used with the straight-away plug will be described in detail.

The oil is contained in a rectangular tank (Figs. 5 and 6), $41 \times 18 \times 13$ inches. The last dimension is the depth. The tank is of sheet iron, galvanized after construction. It is heavily lagged, on the four vertical sides and the bottom, with asbestos, and when running, the top is completely covered with an asbestos cover about $1\frac{1}{2}$ in. thick. The two long vertical sides and the bottom are lined on the inside with a series of 8 German-silver heating coils (C_1, $C_2 \ldots C_8$, Fig. 5); each coil takes about 300 watts at 110 volts. At each end of the tank is a small circular aperture (not shown) ordinarily closed with a flange. Removing the flanges, after draining the oil from the tank, makes it

possible to interchange the thermometers without disturbing the set up. The oil, of which some 35 gallons are used, may be drained into a lagged storage tank (not shown) by opening the cock K_1 (Fig. 5). After the thermometers are interchanged, the oil, still hot, is pumped back into the bath-tank by means of an oil-pump (not shown). The oil that has been used is a heavy tempering oil, quite viscous at room temperature, and having a very high flash-point.

In all of the work here reported, the control of temperature has been by means of a mercury-in-glass thermostat of the familiar type, operating by means of a platinum contact, through two relays, to open or close the circuit supplying the heating coils. Owing to the rather large current (25–30 amperes) carried by this circuit, it was necessary to use alternating current, to avoid troubles with arcing on the make and break. The type of thermostat mentioned has been used with entire satisfaction at temperatures up to 220° C., and, in some earlier work, with fair satisfaction at 240° C. At higher temperatures, the operation is likely to be uncertain because of evaporation of the mercury.

The secondary superheater (S_2, Fig. 5) in which the final adjustment of temperature is made, consists of eight $\frac{3}{8}$ in. copper pipes bent back upon themselves as shown, and silver-soldered at both ends into manifolds. The pipes are arranged in two groups, each group consisting of four pipes in parallel; these two groups may, by suitable arrangements, be placed either in series or in parallel, thus making it possible to double the velocity of steam flow. In the earlier form of the apparatus, with the vertical or U-type, and also with the V-type, of plug case, the secondary superheater consisted of a single coil of $\frac{3}{8}$ in. copper pipe, the total volume of which (0.052 ft.^{3}) was not very different from that of the copper pipes of S_2. The steam velocity in this earlier superheater was necessarily much greater than the velocity of flow in the talc-lined tube T, Fig. 1, or the tube T of Fig. 3, where the diameter of the circular cross-section was about 1 in.; as a result of this, the temperature of the steam at the high side thermometer was higher than that of the oil, by an amount depending on the flow, because of the decrease in the kinetic energy of the steam as it passed from the small cross-section of the superheater to the larger cross-section of the tube T. It was chiefly in order to avoid this defect, with its obvious possibilities, direct and indirect, that the parallel arrangement of pipes was adopted in the later form of superheater. The velocity of flow here is very nearly the same as in the tube T of the plug case (Fig. 3), or as in the conical annular space surrounding the plug, (Fig. 4), when the two groups of superheater

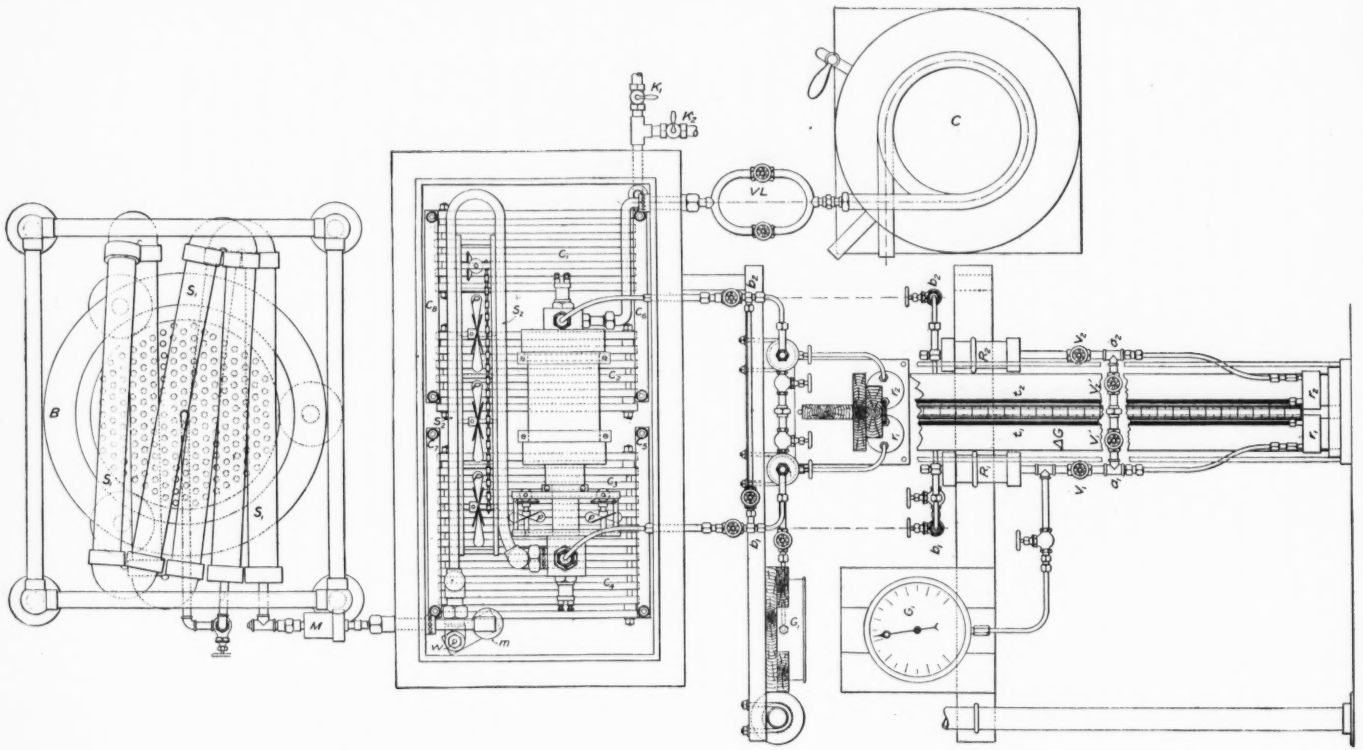

FIGURE 5. General view of main parts of apparatus. Plan and end elevation.

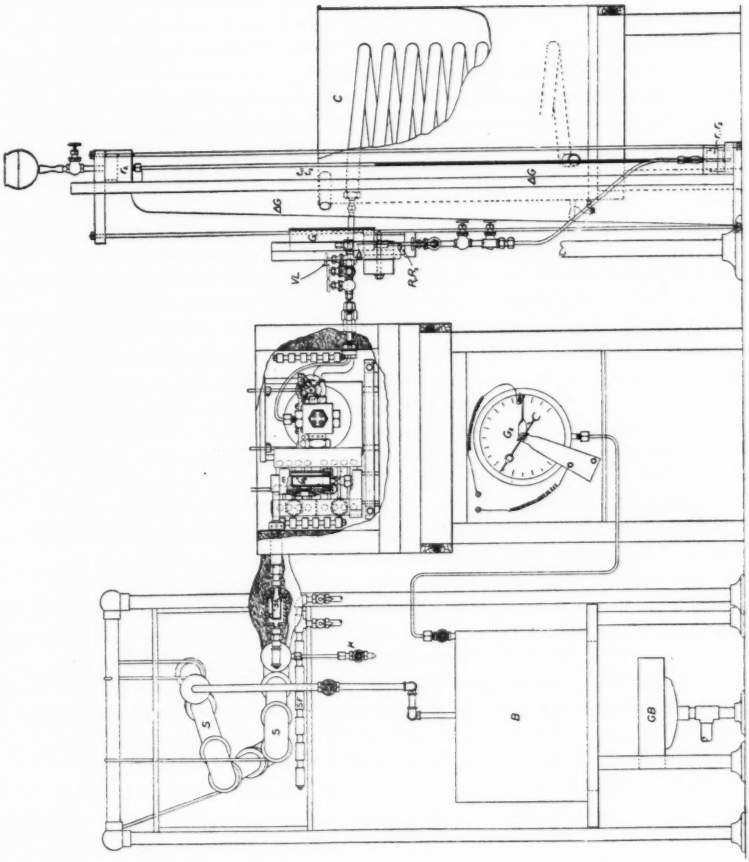

FIGURE 6. General view of main parts of apparatus.— Front elevation.

pipes are in parallel; in fact, for a considerable area of the (p, T) plane, either velocity is so small, for the highest flow rates used, that the steam could be considered at rest so far as any temperature effect of its kinetic energy is concerned. By altering the connections of the superheater so that the groups of pipes are placed in series instead of in parallel, tests of the effect of different velocities in the superheater and in the plug case may be made.

In the operation of the apparatus, it has been customary to bring the steam to a temperature exceeding the high side working temperature by from five to fifteen degrees C., or even more, before it enters the secondary superheater. This superheater thus really acts as a cooler. The chief purpose of this method of operation is to avoid the possibility of the introduction of moisture into the steam by condensation in the short length of heavily lagged pipe connecting the primary superheater (S_1, Fig. 5; S, Fig. 6) with the secondary superheater. (The lagging on this pipe is shown in Fig. 6, but is omitted in Fig. 5). A well (W, Fig. 5) for a mercury thermometer is provided at the entrance of the secondary superheater, to facilitate this regulation of temperature.

As a further precaution against moisture, the steam passes through a strainer (contained in the chamber M, Figs. 5 and 6) just before entering the secondary superheater, and through another strainer (chamber m, Figs. 5 and 6) just after entering it. These strainers are both of alundum and are, in fact, nothing but small editions of the plug in which the throttling takes place. By omitting either or both, tests of the need for them and of their efficacy may be made. Some remarks and conclusions, from experimental results, on the utility of such strainers will be found in section IV, 2 g, which deals with the data derived from the type U plug case.

The stirring devices are indicated at P, P, P, p, p, Fig. 5. The three larger four-bladed propellers, P, P, P provide the main circulation, which is across the lower four pipes of the superheater, under and around the plug case, and back across the upper four pipes of the superheater. Axial circulation is provided by the two smaller four-bladed propellers p, p.

2. Boiler, Primary Superheater and Condenser.

The steam is generated in an 18 inch Stanley automobile boiler (B, Figs. 5 and 6), designed to operate at from 500 to 600 pounds per square inch and tested to 1800 pounds per square inch by hydraulic

pressure. Heat is supplied to the boiler by a gas burner beneath it (*GB*, Fig. 6). Special water level indicators are necessary at the higher pressures, gauge glasses being impracticable. Two different types have been used, one being that regularly supplied by the makers of the boiler. This device, which is very reliable, indicates whether the water level is below or above a not very sharply defined position, but requires a rather large variation in level for its operation. It was found desirable to have a sharper regulation of water-level than this appliance affords, because of the danger of introducing moisture into the steam if the level is raised too high. To secure this, a float enclosed in a vertical steel tube has been used. The float is of glass, copper-plated and then silver-plated. It completes an electrical circuit through a relay and thereby turns off an otherwise lighted incandescent lamp, when the surface of the water is raised to a certain definite level. The silver-plating is necessary to secure reliability of contact; it was found necessary also to silver-plate the inside of the containing tube. The only difficulty with this appliance arose from the gradual penetration of the water to the interior of the float through small flaws in the metal-plating — a rather striking example of the slow solvent action of hot water upon glass.

Pressure regulation is obtained by means of the hack-gauge G_2, Fig. 6. The pointer of this gauge makes an electrical contact between platinum terminals when the boiler pressure rises to the desired value. The completion of this circuit opens, through a relay, an electro-magnetically operated valve which admits air under pressure to one side of a U-tube containing mercury. The ensuing rise of the mercury level in the other side of the U-tube shuts off a portion (or, if desired, all) of the supply of gas to the burner. The electro-magnetic valve closes again when the pressure has fallen sufficiently to permit the platinum contacts mentioned to separate.

As is indicated in Figs. 5 and 6, the steam passes from the boiler to the primary superheater (S_1 in Fig. 5; S in Fig. 6). This superheater consists of six 30-inch lengths of two-inch double strength wrought iron pipe, joined in series by cast-steel return bends, the whole capped at each end with cast-steel caps, tapped to fit the smaller entrance and exit pipes. The pipe connecting the boiler and superheater is heavily lagged, and is heated, throughout its length, by the hot gases from either the boiler furnace or the superheater furnace *SF* (Fig. 6). The superheater and its furnace are enclosed in an asbestos box, which is shown in part in Fig. 6, but not in Fig. 5. The total volume of the primary superheater is about 0.33 ft.

Each length of pipe in this superheater is filled from end to end with a system of copper baffle-plates arranged as shown in Fig. 7. It will be seen that this device imparts to the steam a rotary motion, the direction of which is reversed at each baffle. This insures thorough mixing and the projection of particles of moisture against the hot walls of the pipe. The baffle plates fit snugly into the pipe and make

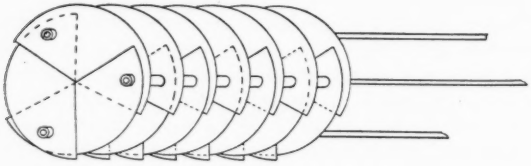

FIGURE 7. Arrangement of baffling plates in primary superheater.

good thermal contact with the walls. To provide for the elimination of such moisture as may be separated from the steam by the action of this series of baffle plates, the drip-cock K (Fig. 6) is placed at the end of the superheater nearest the throttling apparatus. This cock is normally slightly opened during operation.

The furnace for the superheater consists of six lengths of $\frac{3}{8}$ in. pipe, screwing into manifolds at each end. It is fed with gas and air at each of two opposite corners. A uniform fire is secured by independent control of fuel and air for each corner. The furnace is shown at SF, Fig. 6; it is omitted from Fig. 5, to avoid confusion.

After leaving the plug, the steam passes through two valves in parallel (VL, Figs. 5 and 6) to the condenser (C, Figs. 5 and 6). The use of two valves permits a nicer adjustment of pressure drop than is possible with a single valve, and, by halving the valve throttling, makes it possible to operate at a lower mean pressure without the use of an air pump on the condenser. The condenser consists of a helical coil of 1 in. wrought iron pipe in a galvanized sheet iron containing tank, through which passes a continuous supply of cooling water. The condensed steam is received in glass jars and weighed.

3. PRESSURE MEASURING APPARATUS.

The high side pressure is measured by means of a precision Bourdon gauge (G_1, Figs. 5 and 6), made by the Crosby company, and of the

type known as 'standard test gauge.' The gauge was calibrated at the company's factory.

The drop of pressure through the plug is measured directly on the mercury manometer ΔG, Figs. 5 and 6. As is indicated in Fig. 6, this manometer is so arranged that all joints are packed against water rather than against mercury. The two vertical glass tubes t_1, t_2 (barometer tubing) extend to the bottom of the separate reservoirs r_1, r_2. These reservoirs are each filled to about one-fourth of their heights with 500 gms. of mercury. The remainder of the volume of each contains water; the top reservoir r_3, Fig. 6, together with the space in the glass tubes above the mercury columns, also contains water. As the figure shows, the reservoirs r_1 and r_2 are connected respectively to the high and low sides of the plug by means of flexible copper tubing, through the reservoirs R_1 and R_2. The purpose of these latter reservoirs is partly to provide a considerable mass of comparatively cool water, all of which must pass through a constricted passage before any steam can escape, in the event of the bursting of a glass manometer tube. The time required for blowing out this reserve of water is sufficient to enable the experimenter to close the necessary valves, thus preventing further damage to the manometer or other apparatus near it by reason of the presence of hot steam or hot water. The 'constricted passages' referred to are at the bottom of each of the reservoirs R_1 and R_2, where the diameter of the passage is reduced to about 0.04 inch for a length of about 0.25 inch. These constrictions also serve to damp oscillations of the mercury in the manometer tubes.

A further purpose of the reservoirs R_1, R_2 is to prevent variations in the amount of water in the connections from the gauge to the plug from affecting the registered pressure difference. Such variations may be due to gradual and undetected condensation or to a sudden change in the pressure difference, which would force the water down in one connection and up in the other, or to other causes. Whatever the origin of these variations, the portion of either connecting passage first affected is that between the oil bath and the reservoirs R_1, R_2; as this part is nearly level, the effect on the registered pressure difference of removing even a large part, or all, of the water in it, is slight. If the change in the amount of water is great enough to extend beyond this portion of the connection, the large area of cross-section of the reservoirs R_1, R_2 prevents even a large change in the volume of the contained water from greatly affecting its level. It may be stated that the connections between the plug and the differential gauge

shown in Figs. 5 and 6 were adopted only after a considerable amount of experimenting, and after other forms (in particular, one in which a part of the tube between either plug connection and the corresponding reservoir was vertical) had proven themselves unsatisfactory. With these earlier forms, it was not found possible, even with the most careful attention to filling the entire connections with water before starting a run, to prevent the formation of troublesome bubbles of air or steam in the parts of the tubes lying between the reservoirs and the plug. The presence of these bubbles was indicated by a failure of the mercury columns to equalize in height on shutting the low side flow valve VL; they were localized in the parts of the connections mentioned by opening the test valves V_1', V_2' in the cross-connection $a_1\ a_2$ (Fig. 5) and closing one of the valves V_1, V_2, thus applying the pressure existing on one side of the plug to both sides of the gauge through the parts of the connections lying between the cross-connection $a_1\ a_2$ and the gauge. No differences of level of any significant amount were ever observed when this was done.

The upper cross-connection $b_1\ b_2$ (Fig. 5) did not exist in the earlier types of the connections between the plug and the manometer, but was added when the type shown in Fig. 5 was adopted. It was added because it was found that a part of the difference in level of the mercury columns which persisted after closing the valve VL could not be ascribed to the presence of air or water vapor in the upper connections, but was due to a lag of the equalization of pressure on the two sides of the plug behind the more or less regular pulses of the high side pressure caused by the action of the piezostat: the throttling in the plug, particularly in the case of radial flow plugs, prevented immediate equalization of the pressure following a pulse, and the upper cross-connection, by short-circuiting the plug, removes the cause of this lag. All these refinements in what would seem to be a comparatively insignificant part of the apparatus were necessary because the difference of pressure which persisted after stopping the flow of steam had to be applied as a correction to the pressure difference observed during a run. With the earlier forms of connections, this correction was relatively large, being frequently in the neighborhood of 0.5 cm. of mercury and occasionally as much as 1.0 cm. or even more. It was, moreover, uncertain, being frequently different at the end of a run or a series of runs from what it had been at the beginning. With the type of connection shown in Fig. 5, the correction was reduced to so small a figure that it could almost be ignored, for it now ranged from zero to 1 or 2 mm. of mercury at the most (averaging about 0.8 mm.) —

an error of little consequence in total pressure drops of from 40 to 130 cm. of mercury when the errors due to other causes are considered.

The orifices in the bushings which screw into the plug case, and through which connection is established with the manometer, were covered with several circular discs of copper gauze, capped and held in place by a similar disc of sheet copper fastened to the bushing with screws. The gauze discs and the plate were of a diameter about three times that of the orifice, and they were located concentrically with the orifice. The purpose of this arrangement was to avoid Pitot tube effects. It has been used before by other experimenters.

The possibility of moisture entering the apparatus through the pressure connections is obviated by the shape of the connecting tubes inside the oil bath. (See Fig. 6.)

The possibility of an error in the measurement of the difference of pressure due to the fact that the pressure at the bulb of the thermometer may not be that at the point at which the connection is taken off from the plug case to the manometer, was carefully investigated. It was found that, if such an error exists at all, it is of inappreciable magnitude. The method employed in settling this question consisted in measuring the difference in pressure between these two points directly on an auxiliary manometer while the pressure difference on the two sides of the plug was maintained at various values such as were commonly used in regular runs, and further, in making rapid and large changes in the rate of flow while noting the resulting changes in the pressure difference registered by the auxiliary manometer. The actual conditions of a regular run were very closely imitated by using a dummy thermometer as the means of impressing the pressure existing at a thermometer bulb in a regular run on one side of the auxiliary manometer. The observations were made on both sides of an axial flow plug such as is shown in Fig. 1. The readings of the auxiliary manometer were small, averaging 0.09 cm. Hg. on the high side and 0.07 on the low side. On the high side, all readings happened to be zero or of the same sign, which was not that to be expected from the physical circumstances of the case. On the low side, differences of both signs as well as zero were observed. When the main pressure difference was varied rapidly from 20.8 cm. to 86.5 cm., no change was noted in the reading of the auxiliary manometer, although its reading changed by 0.20 cm. when the main pressure difference was brought back to 20.8 cm. There is no doubt that the actual differences registered by the auxiliary gauge were accidental and due to variations in the water level in the connections thereto.

4. Thermometers and the Measurement of Temperature.

a. Description of thermometers.

All temperature measurements in this work have been made by means of platinum resistance thermometers. A large amount of experimenting, the details of which are not of much interest, was done before a satisfactory type of thermometer was evolved. The requirements of a thermometer suited to exact calorimetry at fairly high temperatures and pressures are rather severe. The chief ones may be said to be: (*a*) mechanical durability, (*b*) permanence of calibration, (*c*) ability to follow fluctuations of the temperature of the surroundings with reasonable rapidity, (*d*) similarity in all respects of the two members of a pair of thermometers. The last-named is particularly important where thermometers are to be used differentially, because the advantage of the differential method obviously decreases as the difference in the resistance of the two thermometers at the same temperature increases.

The type of thermometer which was finally adopted and used is illustrated in Fig. 8. It is essentially an adaptation of the calorimetric

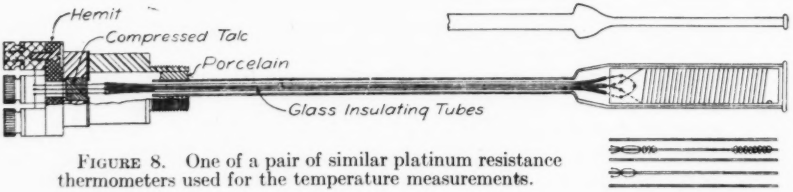

Figure 8. One of a pair of similar platinum resistance thermometers used for the temperature measurements.

thermometer of Dickinson and Mueller,[4] but is naturally much more substantial mechanically, and much less sensitive. It is of the Callendar type, the leads being of No. 26 copper wire, silver soldered at one end to the platinum, at the other end to copper binding posts. In the instrument shown, the binding posts are silver-plated. It was found desirable to do this and also to silver-plate the lugs at the

4 Bulletin of the Bureau of Standards **3**, 641 (1907).

thermometer ends of the leads which connect the thermometers to the bridge, in order to reduce contact resistance to as low a figure as possible. One thermometer, with flat leads similar to those of the Dickinson-Mueller thermometer, enclosed in a flat tube, was constructed, but difficulty in securing adequate insulation resistance was encountered, and the type was abandoned. The bulb and stem, which in certain thermometers were separated by a layer of porcelain, are of steel, the stem being $\frac{1}{4}$ inch in diameter and having a wall thickness of about 0.03 inch. The stem is secured to the head by means of a porcelain joint. The bulb is flat, about $\frac{5}{8}$ inch wide, and with a space of about $\frac{3}{32}$ inch between the two flat walls inside. The length of a thermometer from the bottom of the bulb to the upper porcelain joint is about $9\frac{1}{2}$ inches.

Certain experiments with these thermometers, having for their object the investigation of the magnitude of various errors in the measured difference of temperature on the two sides of the plug, will now be briefly described.

b. Effect of conduction of heat down the stem of the thermometer.

The porcelain joints which have been mentioned were employed for the purpose of minimizing any error which might arise from this source. As nothing was actually known regarding the possible magnitude of this error, it was considered desirable to make some experimental tests on it. To this end, a thermometer having two porcelain joints, one between the bulb and the stem and one between the stem and the head, was used. Two sets of clamps, made of copper and soft iron, were constructed: one set of clamps could be applied to the upper joint so that it short-circuited the heat-insulation afforded by the porcelain; the other set could similarly be used to short-circuit thermally the lower joint. Both sets were carefully made to fit the joint concerned closely and to give good thermal contact with the metal on each side of the joint. Comparative readings of the thermometer when immersed in the vapor of boiling naphthalin, exactly as for calibration at this point, were made, with the following different arrangements: (i) neither joint short-circuited; (ii) both joints short-circuited; (iii) upper joint free, lower short-circuited; (iv) lower joint free, upper short-circuited. Taking (i) as standard, case (ii) lowered the apparent boiling point by 0°.01 C; case (iii), by

0°.02 C; case (iv), by 0°.005 C. While the relative magnitudes of these results are hardly such as would be expected, the fact that a depression of the apparent temperature was obtained in each case indicates the existence of a real error from the cause under investigation and that the porcelain joints are of some value in diminishing its magnitude. The experiments were not refined, as will be inferred from the method used. They were carried no further than was necessary to secure the information desired. They can hardly be regarded as conclusive regarding the size of the error to be expected if no thermal insulation is used, for it is difficult to calculate even approximately the relative insulating effects, as regards the bulb, of a porcelain joint and of the long, thin stem. Without joints, or even with them, the temperature gradient in the stem must be concentrated near its upper end, and the insertion of an insulating joint at this point may be expected to have a much larger effect on the effective thermal conductance of the combination than would be calculated on the assumption of a uniform temperature gradient longitudinally in the stem and radially in the joint. However, it is at least reasonable to infer from the above experiments that the error in the *measured temperature drop through the plug* due to the cause we are considering would be of insignificant magnitude, whether insulating joints are used or not. For such an error would be that occasioned by conduction down the stem of the low side thermometer, with a temperature difference between the head and the bulb not exceeding 5°, while in the above experiments this temperature difference was at least 140°, since it was observed that naphthalin would freeze on the head of the thermometer. On the basis of the above data, the omission of insulating joints could then affect the measured difference of temperature by not more than $0.02 \times \frac{5}{140} = 0°.0007$, and if we assume that the effective thermal resistance of the stem is even as much as 10 times that of the insulating joints, conduction down the stem of a thermometer without joints would produce an error in the temperature difference of only 0°.007, which is not greater than arises accidentally from uncontrollable causes.

The thermometers used in all the work on the Joule-Thomson effect described in this paper have the upper porcelain joint, but not the lower one. It is difficult to get an absolutely steam-tight joint, and even a small leak at the bulb would obviously be disastrous, while such a leak at the upper joint would be of much less moment. Besides, if porcelain joints are to be used at all, the upper one is probably of more value than the lower.

c. Radiation effects.

In a further study of the question of heat-conduction down the walls of the thermometer tube, undertaken about a year after the experiments just described, some unexpected effects due to radiation were noticed. These experiments were made with a specially-constructed thermometer, the coil of which is illustrated in Fig. 9. As is indicated,

FIGURE 9. Coil of the experimental platinum resistance thermometer No. 8.

the coil is wound on a mica cross supported by means of four quartz tubes which also serve to insulate the leads. The containing tube, of uniform bore throughout and large enough to permit the coil to slide in or out easily, may be of either steel or glass. No insulating joints were used. This thermometer is called No. 8 in what follows.

The experiment consisted in inserting this thermometer, together with a comparison thermometer (the one used in the experiments just described, hereafter called No. 4) in the vapor of naphthalin boiling in the hypsometer (see below, page 762) regularly used for comparing thermometers. Several measurements, extending over twelve days, were made with both the steel tube and the glass tube. The apparent temperature by No. 8 was invariably *higher* when the steel tube was used. Omitting one or two measurements, doubtful because some naphthalin penetrated to the interior of the glass tube before a proper method of packing it was discovered, the results were as follows, in chronological order:

TABLE I.

Experiment No.	Kind of tube	$R_8 - R_4$ Ohms	°C*
1	glass	0.06833	3.578
2	glass	0.06820	3.571
3	steel	0.06913	3.620
4	steel	0.06887	3.606
5	glass	0.06824	3.573
6	steel	0.06891	3.608

* This is the temperature equivalent of R_8-R_4 on the scale of 4. It would be slightly different on the scale of 8, but the hundredths of degrees would be the same on the two scales. It is of course immaterial which scale the difference is referred to, as far as the comparison is concerned.

The mean apparent *rise* in the temperature when the glass tube is replaced by the steel tube is 0°.037 C; as will be seen, the difference is quite uniform. Being in the direction opposite to that of the difference which could be caused by conduction along the tube, it was attributed to radiation. To test this, a further measurement was made with the inside of the glass tube blackened to resemble the inside of the steel tube. The result was $R_8 - R_4 = 0.06895$ ohm = 3°.610 C., which is practically the value obtained with the steel tube. In addition to verifying the suspicion that the difference was due to radiation, this result substantiates the conclusions reached in the experiments described under *b* regarding the effect of heat conduction along the thermometer tube. The steel tube of No. 8 was of twice the diameter of the stem of No. 4 and also somewhat thicker, having an area of cross-section perhaps three times as large as that of No. 4.

The surprising magnitude of this radiation error was sufficient effectually to discourage the use of thermometers of the type of No. 8 in the work of the main research. If it is thought that it ought also to arouse suspicion as to the existence of a comparable error in the case of the thermometers actually used, it may be replied that the coil of 8, wound in contact with quartz tubes of high reflecting power and separated by a comparatively thick layer of air from the walls of the containing tube, is necessarily far more susceptible to radiation effects than the coil of a thermometer of the type of No. 4, wound on a strip of mica and fitting snugly into its flat tube, from either side of which it is separated by only a thin layer of mica. Moreover, such radiation error as actually exists must be practically the same for both high side and low side thermometers, and hence without effect on the *difference* of temperature.

d. Effect of heat conduction along the thermometer leads, and of unequal temperature distribution in these leads.

To test this, a wing of thin sheet copper was attached to one of the binding posts of the coil circuit of thermometer No. 8 during an ice point calibration. The apparent elevation of the freezing point observed when this wing was heated to redness to within one inch of the binding post was 0.00102 ohm = 0°.53 C. The heat was sufficient to oxidize the copper binding post very appreciably. While the magnitude of this difference seems disturbingly large at first glance, a simple calculation shows that the actual effect in the case of a measured difference in temperature in a Joule-Thomson experiment would

be insignificant. In such an experiment, the temperature difference existing between the bulb of the thermometer and the binding-posts at the head is not, as a rule, in excess of 5° C., whereas, in the above test, this difference must have been at least 200° C. This would signify nominally an error of 0°.013 in the measured temperature difference; but in the above-described test, only one of the four binding-posts was heated, so that the ordinary function of the compensating circuit was wholly inoperative. This is of course not the case with the Joule-Thomson experiment, where we may reasonably expect the nominal difference of 0°.013 to be eliminated to within 10 per cent. of itself at most.

To test the effect of inequalities in the temperature of the leads connecting the thermometers with the bridge, large differences of temperature in the coil-circuit and compensating-circuit leads were artificially created. Several tests of this general nature showed that no error from this cause need be feared with the leads as arranged under normal operating conditions. The leads used were of No. 12 solid copper wire, about 22 feet long; resistance of a pair, about 0.037 ohm.

e. Contact resistances.

With unplated copper binding posts on the thermometers, it was found necessary to keep them and the lugs attached to them carefully polished, and to set up tightly on the binding post nuts, in order to be sure that the net contact resistance might not exceed $0.0_4 44$ ohm (0°.0022 C), the limit of sensitivity of the bridge. With silver-plated binding posts, much less care was necessary, and the comparatively small amount of work needed to put on a thin plating of silver is amply repaid by the abolition of the continual necessity of fussing over contacts and by the additional security felt regarding experimental results.

f. Variations in lead-resistance.

It is necessary in work of this sort to keep track constantly of the resistances of the leads from the apparatus to the bridge, particularly if all or portions of these are of stranded wire, as is sometimes necessary for the sake of flexibility. Mere bending of a stranded lead will sometimes produce a noticeable change in its resistance. An unnoticed parting of a strand will frequently cause a very appreciable

change in the lead resistance. Solid leads are much to be preferred, wherever it is possible to use them.

g. Calibration of differential thermometers.

The measurement of a change of temperature in continuous flow work involves the use of two thermometers, which must be as nearly alike as it is possible to get them, if the benefits of a direct measurement of the difference of their resistances (as against the separate measurement of the resistance of each) are to be completely retained. Since it is not possible to make two thermometers exactly alike, there will always exist a difference in their resistances when they are at the same temperature. It will be convenient to speak of the combination of two thermometers used in this way as a *differential thermometer*, and to call the difference in their resistances at a common temperature the *normal resistance* of the differential thermometer. A particular differential thermometer will be referred to by means of a number made up of the numbers of the component thermometers, the number of the component thermometer having the higher resistance coming first: e. g., 67 is the differential thermometer composed of the individual thermometers 6 and 7. The normal resistance of this thermometer will be denoted by R_{67}, etc.

In order to get the difference of temperature corresponding to a measured difference of resistance, the normal resistance of the differential thermometer must be eliminated from the latter. This may be done by calibrating the differential thermometer, determining its normal resistance as a function of the temperature, and subtracting from a measured difference of resistance the normal resistance of the thermometer as calculated for the temperature in question; or, the experiment in which the measured difference of resistance is obtained may be repeated with the component thermometers interchanged. In most of the work described in this paper, both methods have been used; but, except where for some reason the interchange method was not used, the actual elimination has been effected by this method, the calibration serving merely as a check. There is little evidence to indicate that the value of the normal resistance as given by one of these methods is different from that given by the other; thus, the mean of six values of (R_{67} by calibration $-$ R_{67} by interchange) at 165° C. is 0.00003 ohm or about $0°.001_5$ C. with an average deviation of 0.00025 ohm or about 0°.012 C. The time interval involved in this work is about a year and a half. Of the six calibrations of the

differential thermometer involved, two were made necessary by reason of accidents necessitating repairs to the thermometers which involved changes in their constants. Each of the values of R_{67} (a single one excepted) by interchange is the mean of from three to five determinations with the same set-up of apparatus, but with varying flow rates. The greatest average deviation involved in any of these five sets of interchange values of R_{67} was 0.00018 ohm, (0°.009 C.); the least, 0.00007 ohm (0°.003$_5$ C.); the mean, 0.00010 ohm (0°.005 C.). Thus the interchange method gives results which are more consistent among themselves than they are with the results obtained by calibration. It cannot be supposed, however, that this signifies a real difference of the indicated magnitude in the values of the normal resistance by the two methods, for the reason that uncontrollable fluctuations in this resistance between calibrations are at least as great as the discrepancy between its values as obtained by interchange and by calibration. The size of these fluctuations is shown in Fig. 10, which exhibits the behavior of R_{67} at 0° C. over a period of eighteen months. It is partly because it is not always convenient to calibrate the thermometer immediately before or after (or both before and after) a set of runs that the interchange method has been used to eliminate the normal resistance; partly also because, by this method, the thing eliminated is the normal resistance under actual conditions of use. This last reason was of more weight before experience had proven that the use of the differential thermometer under pressure did not affect its normal resistance, as conceivably might have been the case: in calibration, of course, the normal resistance is determined at atmospheric pressure.

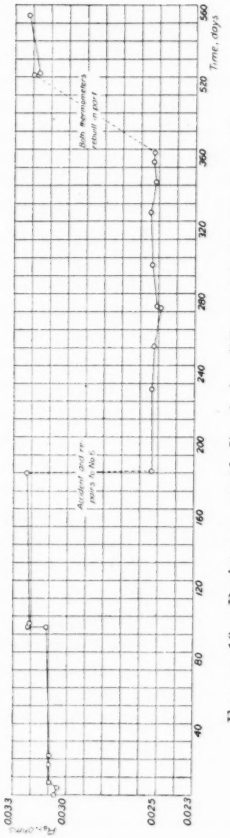

FIGURE 10. Resistance at 0° C. of the differential thermometer 67.

In spite of the fact that the calibration is not, in general, used directly in calculating the temperature difference from the resistance measurements, it is distinctly worth while, in the writer's opinion, for the following reasons:

First, it gives a valuable check on elimination by interchange; second, it affords a very useful indication of the precision of the calorimetric work; for the value of the normal resistance as determined by interchanging thermometers in a Joule-Thomson experiment includes the accidental errors of the experiment, so that the divergence of this value from the calibrated value is an index of the magnitude of these errors; third, if through some accident or oversight in setting up the apparatus one of a pair of runs has to be rejected, the other may still be used; for example, of a certain set of nine runs, the first four (with thermometer No. 7 on the high-pressure side of the plug) had to be thrown out because of a fault in one of the leads from the plug to the bridge which was not discovered until after these runs had been completed. The value of the remaining runs, with No. 6 on the high side, was not affected, because a calibration made immediately after afforded the means of eliminating the normal resistance.

In calibrating the differential thermometer, the chief difficulty lies in getting the two thermometers simultaneously at the same temperature when this temperature exceeds that of the room by say 150° or more. At least two such points are necessary. It is probable that the most satisfactory method of obtaining them would be to employ a double oil bath, provided a sufficiently sensitive thermostat for the higher temperatures could be devised. The method employed by the writer made use of a large, specially-constructed hypsometer of the well-known Regnault type, in which the two thermometers could be exposed to the vapors of substances which boil at convenient temperatures. The hypsometer was heavily lagged with asbestos except for the reservoir containing the liquid and for an air condenser by means of which most of the vapor was liquefied and returned to this reservoir. The substances used in this apparatus were water, cumol (about 165° C.), naphthalin (218° C.), diphenylamin (310° C.). Water, of course, gives no trouble whatever. Cumol, in spite of its comparatively low boiling point, has always been unsatisfactory, chiefly because its boiling point rises continuously as vaporization progresses, even if the liquid has been previously distilled. Naphthalin is very satisfactory. Diphenylamin has been little used, as its boiling point lies beyond the temperature range required in the work discussed in this paper. It has the same disadvantage as cumol as regards

rise of the boiling point during vaporization, though in a lesser degree.

It is doubtful, however, whether the hypsometer method of calibration would be satisfactory at temperatures as high as that of boiling diphenylamin, even if the boiling point were perfectly steady. It was found early in the work that even when naphthalin was used, the temperature in the region occupied by the two thermometer bulbs was not uniform, but that the apparent value of the normal resistance of the differential thermometer could be changed by an amount of the order of one or two hundredths of a degree on the scale of either thermometer by merely interchanging the positions of the thermometers. This effect was not accidental, but could be obtained repeatedly with great uniformity and definiteness; because of this, it could with confidence be regarded as eliminated by taking the mean of a number of measurements equally distributed between direct and reversed positions of the thermometers. To facilitate this method of observation, which was always used, the top of the hypsometer, from which the thermometers were suspended, was made capable of rotation about a vertical axis, with respect to which the two thermometers were symmetrically arranged; thus the thermometers could be made to exchange positions without withdrawing either from the hypsometer or disconnecting it from the bridge.

This non-uniformity in temperature of the vapor column in which the two thermometers were immersed was first noticed in a much simpler comparator than that which has been described. This earlier comparator consisted merely of a large glass tube (3 inches in diameter) of uniform bore, closed at one end and lagged on the sides with asbestos. With this apparatus, the apparent normal resistance of the differential thermometer could be changed by an amount equivalent to 0°.16 C. by merely interchanging the positions of the thermometers,— which signifies a difference of 0°.08 C. in the temperature of the vapor (naphthalin) at two points at the same level and about 1.5 inches apart. This difference was not accidental; in an experiment lasting over three hours, the apparent differential resistance in either position of the thermometers remained constant to within less than 0°.01 C. The only evident asymmetry of arrangement which might account for this rather remarkable effect consisted in the fact that the thermometer bulbs were always on a line perpendicular to a brick wall near which the apparatus was set up; but the phenomenon was not in the least altered by the use or omission of radiation shields around the thermometer bulbs, and has been noticed, in the Regnault comparator, even when the above-mentioned asymmetry was avoided.

The results of the calibration of a differential thermometer are expressible in the form of a relation quadratic in the temperature or in the resistance of one of the component thermometers. As far as convenience in computation of temperature differences is concerned, there is not much to choose between these two methods of representation. As a check on the accuracy of the latter, three differential thermometers, made up of thermometers 4, 6 and 7, were calibrated at the ice, steam and napthalin points, and the differential thermometer 67 was also calibrated at the boiling point of cumol. The relation $R_{64} = R_{67} + R_{74}$ should hold for all temperatures. The observed values of the three resistances satisfied this relation to within 0.00013 ohm (0°.0065 C.) at the steam and naphthalin points, and to within 0.00006 ohm (0°.003 C.) at the ice point. These results may be regarded as satisfactory, since the closing errors are to be divided among the three thermometers. After forcing the closure at each point, the following equations were obtained:

$$R_{67} = 0.03071 + 0.000532\,(R_7 - 5.3003) - 0.0003554\,(R_7 - 5.3003)^2$$
$$R_{74} = 0.01823 + 0.027436\,(R_7 - 5.3003) - 0.0000354\,(R_7 - 5.3003)^2$$
$$R_{64} = 0.04894 + 0.028009\,(R_6 - 5.3310) - 0.000396\,(R_6 - 5.3310)^2$$

From these one finds at the boiling point of cumol (165°.7, $R_6 = 8.748$, $R_7 = 8.720$):

$$R_{67} = 0.02837,\ R_{74} = 0.11163,\ R_{64} = 0.13992$$

so that the closing error is 0.00008 ohm (0°.004). The observed value of R_{67} at cumol was 0.02842 ohm. Similarly one finds, for example, at 130° C. ($R_6 = 7.998$, $R_7 = 7.969$)

$$R_{67} = 0.02960,\ R_{74} = 0.09120,\ R_{64} = 0.12082$$

from the above equations: here the closing error is only 0.00002 ohm.

It should, however, be stated that the satisfactory results set forth in the preceding paragraph must be regarded as partly accidental — or, at least, that more care was taken in the experimental work from which they were derived than limitations of time ordinarily permit. Thus three later calibrations of 67 give 0.00033, 0.00021 and 0.00013 ohm respectively for the difference between the observed normal resistance in the vapor of boiling cumol and the value of this resistance as calculated from an equation of the above form in which the constants are determined from observations at the ice point and at the boiling points of water and naphthalin. These discrepancies are undoubtedly due in large measure to the difficulties of the calibration

at the cumol point, where it has always been hard to secure conditions sufficiently steady to warrant confidence in the results. In fact, cumol was finally abandoned altogether because of this trouble.

The difference in magnitude between the coefficients in the equation of calibration of the differential thermometer 67 and the corresponding coefficients in the other two equations is due to circumstances which are worth a brief remark. The two thermometers 6 and 7 were made of consecutive pieces of platinum wire taken from a spool marked 'Heraeus' purest,' and no pains were spared to make these thermometers exactly alike. The two pieces of platinum were annealed at a red heat, and their resistances were adjusted to equality within less than 0.0005 ohm at room temperature. After winding, the platinum was once more annealed and the net resistances of the two thermometers were again equalized by making slight adjustments in the compensating circuits. The resistance of each was also now very nearly that of No. 4. The two thermometers were then inserted in their respective tubes and the caps which close the bottoms of these tubes were brazed on. This operation nullified all the efforts at equality of resistance which had been made. The resistances at zero differed by nearly 100 times as much as before brazing, and each exceeded that of 4. Moreover, as the equation of calibration shows, the normal resistance of 67 *decreases* slightly with rising temperature, although it is so nearly constant over a considerable range of working temperature that the actual temperature need not be known accurately so far as elimination of the normal resistance is concerned; this is a convenience rather than otherwise.

Thermometer No. 4 was made of Baker platinum, and it is therefore not to be expected that it would combine with either 6 or 7 to form a differential thermometer as satisfactory as 67 itself; in fact, both R_{74} and R_{64} vary much more rapidly with the temperature than R_{67}, as is indicated by the equations of calibration. The dissimilarity of 4 to 6 and 7 is further shown by a comparison of the Callendar constants for the three thermometers. The constant δ has the values 1.54, 1.65, 1.58, and the constant $K\left(=\frac{R_{100}-R_0}{100\,R_0}\right)$ the values 0.003795, 0.003859, 0.003882, for 4, 6 and 7 respectively. The value of δ is ordinarily taken as a criterion of the purity of the platinum: if δ exceeds about 1.50, the platinum is regarded as impure. On the other hand, values of K as large as those given for 6 and 7 indicate a high degree of purity in the platinum. There thus seems to be some doubt whether a large value of δ is, by itself, a reliable indication of impure platinum. The

writer has learned in conversation of a thermometer made of the best platinum obtainable, the δ of which was 1.68; and in a paper on the mechanical equivalent of heat,[5] Roebuck describes two thermometers made from the same sample of platinum, for which the δ's were respectively 1.748 and 1.567, with fundamental coefficients of nearly equal magnitudes (0.0035127, 0.0035387).

h. Calibration of individual thermometers.

The temperature scale in all the work here described depends on calibrations in ice, steam and naphthalin vapor. The naphthalin used was Kahlbaum's Reagent, its boiling point being taken as

$$t_n = 218.0 + 0.058\ (H - 760)$$

in which H is the pressure in millimeters of mercury under standard conditions.[6] Ordinary drug-store naphthalin boils at almost the same temperature. The apparatus used for the boiling point of naphthalin was that which has become standard for the sulphur point.[7] Drip-cones of asbestos paper and aluminum radiation shields were regularly used, but little effect was noted when either or both were omitted.

Some care was found necessary at the ice point. A thermos bottle was used in the earlier work, but the apparatus finally found most reliable consisted of a 3-inch glass cylinder, lagged with asbestos, and provided with a funnel-shaped bottom, to which a rubber tube with stop-cock could be attached. The cylinder was long enough to permit immersion to the head of the thermometer. It was found expedient to cool the thermometer bulb and stem in a separate ice-bath before immersion in this ice-point apparatus. Experiments conducted to determine whether there was any difference in the freezing points as given by commercial (natural) ice and by ice made from distilled water showed no difference to the order of the accuracy of the resistance measurements, and the commercial ice was thereafter regularly used, because of the lack of facilities for making the other.

Distilled water was always used at the steam point.

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5 Phys. Rev. (2), **2**, 79 (1913).
6 See Waidner and Burgess, Bull. Bur. St., **7**, 1–9 (1911).
7 See Waidner and Burgess, Bull. Bur. St., **6**, 149–230 (1909–10).

i. Resistance Measurements.

All resistance measurements have been made by means of a Carey-Foster bridge and depend ultimately on Wolff standards certified by the Physikalisch-Technische Reichsanstalt. All contacts on the bridge except those in the galvanometer and battery branches are mercury contacts. The bridge has two slide wires, both of manganin, the finer (0.054 ohm/cm.) being used for measuring the resistance of the high side thermometer, balanced against one or more secondary standard coils. The other wire (0.00445 ohm/cm.) was used in measuring the difference of resistance of the two thermometers, balanced against each other. The galvanometer was sufficiently sensitive to indicate a motion of the slider of 0.1 mm. from the point of balance on the coarse wire, so that the order of the precision of the differential measurements is 0.0$_4$45 ohm, or about 0°.0022 C. The precision of the measurement of high-side temperature, in so far as it depends on the resistance measurement, is about 0°.03 C.

IV. EXPERIMENTAL RESULTS ON THE JOULE–THOMSON EFFECT.

1. Preliminary Discussion. Methods of Eliminating the Effect of Heat-leak.

Before beginning the consideration of the results obtained with the various types of throttling apparatus which have been described, it seems desirable to outline briefly a few points concerned with the general question of heat leakage and the methods which have been or may be used to eliminate its effect from the directly measured ratio of temperature drop to pressure drop. If we call the last mentioned quantity the 'apparent Joule-Thomson effect' and denote it by μ', we shall have

$$\Delta T = \mu \cdot \Delta p - \frac{\delta Q}{f C_p} \tag{1}$$

and hence

$$\mu' = \mu - \frac{\delta Q}{f \Delta p C_p} \tag{2}$$

in which μ is the true Joule-Thomson effect, f is the flow (mass of fluid

passing in unit time), δQ is the heat leakage in unit time (counted positive if heat is received by the fluid), C_p is the specific heat at constant pressure and ΔT and Δp respectively the temperature and pressure drops, assumed to be so small that μ and C_p are sensibly constant over the temperature-pressure interval covered in the experiment.

To eliminate the effect of heat leakage, experiments at approximately the same pressure and temperature, but with different rates of flow, must be conducted. The flow may be varied without varying the temperature drop by using different plugs, and if it may be assumed that the various circumstances which affect the total heat-leakage remain the same for all members of a set of experiments made in this manner, it will be possible to eliminate the effect of heat-leak from the results. A second method of varying the relative magnitude of that portion of the observed effect which is due to heat-leak is to vary the pressure drop and the flow together, using the same plug throughout the set of experiments.

The first of these methods involves more labor and greater experimental difficulty than the other, and although, in the work under discussion, a number of plugs giving widely different flows at the same pressure drop have been used, the second method is the one which has been employed in arriving at a result that is believed to be free from leakage errors.

The application of this method to the elimination of the heat leak effect depends, 1°, upon an experimental fact, and 2°, upon an assumption.

1°. The experimental fact is that when a set of μ''s obtained with a single set-up of the apparatus are plotted against the reciprocals of the corresponding flows, the curve so determined is a straight line, within the limits of experimental error. (One or two possible exceptions to this statement will be noted below.) It follows from this that, within the range of experiment, the leakage term in equation (2) must be of the form

$$\frac{\delta Q}{f \Delta p C_p} = A + \frac{B}{f} \tag{2a}$$

in which A and B are constants. B is evidently the negative of the slope of the experimentally determined straight line just mentioned.

Plots of μ' vs. $\frac{1}{f}$ are shown in Figs. 12 and 13. Table III contains

the data on which these plots are based, together with certain other significant data.

2°. The assumption is that $A = 0$. The confidence one feels in the correctness of this assumption is based on the fact that to deny it is to assert that δQ, the heat leak per unit time, contains a term which is proportional to $f\Delta p$. As will appear later (see equation (4) and Table II) this is the same thing as saying that δQ contains a term approximately proportional to $(\Delta p)^{3/2}$ (or to f^3) in the case of axial flow plugs, and to $(\Delta p)^{7/4}$ (or to $f^{7/3}$) in the case of radial flow plugs. Now the probability of the existence of an appreciable heat leak of this sort is, on the face of things, extremely slight, because ΔT is always at least roughly proportional to Δp (see Figs. 15 and 16) and it is hardly credible that there can be any part of δQ which increases more rapidly — 50 to 75 per cent. more rapidly — than ΔT. In putting $A = 0$ we are therefore not merely making an assumption which seems the most plausible of several alternative ones; we are, on the contrary, unable reasonably to make any *other* assumption.

If A is zero, μ may be calculated from *any* point on the line mentioned above by adding the amount B/f to the ordinate at this point. Obviously the easiest method of doing this is to extrapolate the line to the axis $1/f = 0$. This 'extrapolation to infinite flow' presupposes nothing whatever as to the practicability, or even the theoretical possibility, of obtaining an 'infinite flow' of steam, or any other flow lying outside the experimental range. It is merely a graphical method of arriving at the value of μ which is necessarily involved in the straightness of the line and the absence of any part of δQ proportional to $f\Delta p$.

It is consequently necessary to establish only the fact that the graph is truly rectilinear over any range of observation, however short, in order to avail one's self of this method of elimination. For example, if the entire range of observation included a portion known to be rectilinear, with a piece of curved graph at one or both ends, the latter could be entirely ignored. But as a practical matter, it is possible to be reasonably certain that the graph is really straight over any range only by observing that it is straight, within experimental error, over a reasonably extended range of observation. It is easy to imagine that terms of higher orders in $\frac{1}{f}$ might be present in the right hand side of equation (2a), and that these terms, although having so comparatively insignificant an effect within the range of observation as to escape detection if the range is small, might nevertheless seriously

affect the value of μ obtained by extrapolation. If one actually obtained an experimental curve like that imagined above, with a portion rectilinear within experimental errors and another, curved portion nearer the $1/f$ axis, for example, it would not be possible to convince one's self that the true μ could be obtained by extrapolating the straight part. On the contrary, the extrapolation of the curved line would certainly be expected to lead to the true value of μ, and, if it were possible to perform it, the process would differ in no essential respect from the rectilinear extrapolation — that is, it would be merely a mathematical process, with no implied assumption regarding the possibility of physically realizing the inferred portion of the curve. The true value of μ is reached by the extrapolation because the terms multiplying the first and higher powers of $1/f$ vanish for $f = \infty$; and the true value of μ could equally well be obtained by direct calculation for any $1/f$, since the coefficients of these powers must theoretically be known to perform the extrapolation.

The results obtained by the rectilinear extrapolation of the data obtained with several plugs are certainly considerably too small. This is true of the axial flow plugs A1, A2 and A3 and of the radial flow plugs U1 and U2. (Cf. Figs. 12 and 13.) In all of these cases, the origin of the error must be ascribed to the presence of curvature which either was ignored (because it was impossible to take account of it in the extrapolation), as in plugs U1 and U2, or was not detectable, owing to insufficient range of observation, as in plugs A1, A2 and A3.

It is hardly to be supposed that any line would prove to be absolutely straight, if it were possible to obtain results entirely free from accidental error. Hence, although it is perfectly true that the rectilinear extrapolation is merely a convenient method of obtaining a result which could just as well be obtained by calculations confined to the region of observation if the plotted line were known to be strictly rectilinear, there will exist errors which may be properly called extrapolation errors, in the sense that their magnitude will depend upon the probability that the rectilinearity of the graph is a correct inference from the data, and in the sense that this inference and the value of μ deduced from it are the less certainly correct the greater the accidental errors and the greater the ratio of the range of extrapolation to the range of observation. It is also evident that, of two extrapolations which are alike in the two respects just named, that one for which the variation of the leak effect ($\delta Q/f\, C_p \Delta p$) with the flow is the greater — that is to say, that one for which the slope of the line is numerically the greater — will lead to a less trustworthy value of μ than the other.

Mr. S. A. Moss has pointed out to the writer that if one may assume that the total heat-leak per second, δQ, is proportional to the drop of temperature, ΔT, — certainly an assumption which appears more plausible than that δQ is proportional to the drop of pressure — then a plot of $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ should be straight, and the extrapolation of this plot should lead to the correct value of $\frac{1}{\mu}$. This appears on dividing equation (1) by $\mu \cdot \Delta T$; this gives

$$\frac{1}{\mu} = \frac{1}{\mu'} - \frac{\delta Q}{\mu C_p \Delta T} \cdot \frac{1}{f} \tag{3}$$

which represents a straight line in the variables $\frac{1}{\mu'}$ and $\frac{1}{f}$, provided $\delta Q/\Delta T$ is constant.

A few plots of $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ are shown in Fig. 14.

A moment's consideration of the geometrical relations of the two plots will show that if either is straight, with a slope not zero, the other cannot be straight; that if either is straight and approximately horizontal, the other will be nearly straight, and will also be approximately horizontal; and that if 'best representative straight lines' are drawn through corresponding experimental sets of points on the two plots, these lines will have intercepts on the axis of $\frac{1}{f}$ which will be, approximately, the reciprocals of each other, if the two lines are nearly straight and nearly horizontal. The consequence of this is that where the heat-leak effect varies so little within the range of experiment that the extrapolation seems worthy of confidence, it makes little difference which plot is used. At the same time, the results of extrapolating both lines, even in such a case as this, may be seriously in error; that is, *mere* absence of large slopes in both plots does not signify necessarily that their intercepts are true values of μ or of $\frac{1}{\mu}$; see, for example, the curves U1 in Figs. 12, 13 and 14.

Still another method of exhibiting graphically the results of Joule-Thomson experiments is illustrated in Figs. 15 and 16, in which the observed temperature drop is plotted as ordinate against the pressure drop as abscissa. In the absence of heat-leak, all plotted points should lie on a straight line passing through the origin, and that por-

tion of any ordinate lying between the locus of observed points and a line through the origin having a slope equal to the true value of μ, represents, for the pressure drop in question, the heat-leak per unit mass of fluid, occurring during transit through the plug, divided by the specific heat at constant pressure.

It will be seen from Figs. 15 and 16 that a simple linear equation can express the relation between ΔT and Δp with fair accuracy in most cases. With the further aid of a relation connecting flow and pressure drop, both of these quantities may be eliminated from equation (2), leaving an equation expressing δQ as a function of ΔT only, provided μ is known. Such relations are of some value in a comparative study of heat leakage in different types of apparatus.

The relation between flow and pressure drop which has just been mentioned is expressed quite accurately by

$$f = K\,(\Delta p)^r \tag{4}$$

for the present experiments. Here r is approximately $\frac{1}{2}$ for axial flow plugs and approximately $\frac{3}{4}$ for radial flow plugs. In Fig. 11, logarithmic plots of flow against pressure drop for several plugs are given. Table II gives the constants of the equation (4).

TABLE II.

Constants of equation (4). Flows in kgm./hr. Pressure drops in kgm./cm².

Plug	K	r	Plug	K	r
A 1	18.6	0.462	V 1*	35.2	0.689
A 2	29.8	0.444	V 2*	24.1	0.799
A 3	29.4	0.446	U 1	28.0	0.749
A 4	39.2	0.498	U 2	25.6	0.765
			S	16.4	0.751

2. Discussion of the Experimental Results.

a. General.

In the plots of Figs. 11 to 16 inclusive, the designations attached to the several curves have the following significances:

* The relation (4) is only roughly true for these plugs, owing to choking of the plug while in service. This is very noticeable with radial flow plugs, especially where the steam has not been previously strained through an alundum strainer. The same porous cup was used in all radial flow plugs, and the variations in the constants are due to the gradual 'aging' of the cup. The plugs are listed in chronological order.

1°. A1, A2, A3, A4 refer to four axial-flow set-ups, all similar in general respects to that shown in Fig. 1 and described in Section II, 1. In all of these plugs, there are 37 holes $1\frac{1}{2}$ inches long arranged in a hexagonal pattern at each end of the soapstone block S; in plugs A2,

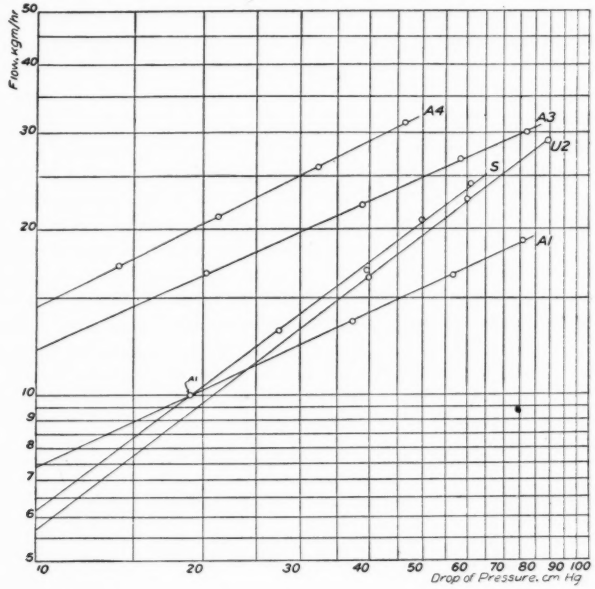

FIGURE 11. Logarithmic plots of flow against pressure-drop. Abscissae, pressure drops in cm. Hg. except for plug S, for which abscissae are one-half of pressure drops in cm. Hg. Ordinates, flow in kgm. per hour.

A3, A4, the diameter of each of these holes is 0.086 inch. In plug A1 the diameter of each hole is 0.076 inch. In plugs A1, A2 and A3, a single piece of solid manganin wire having a diameter of 0.044 inch is threaded through all 74 holes; in plug A4, this is replaced with a stranded invar wire having ten strands, each of about 0.01 inch diameter. The block S' is the same in plugs A1 and A2, in which the cross-channel is about $\frac{1}{4}$ inch in diameter and is located on the axis of the block. The thickness of each of the blocks S and S' is about two inches and the length of the cross-channel is about $4\frac{1}{2}$ inches.

In the block S' of plug A3, the cross-channel is located with its axis on a line about $1\frac{3}{8}$ inches from the lower face of the block. In other respects, the block S' of A3 is similar to that of A1 and A2. The purpose of locating the cross-channel of A3 further from the lower face of the block than in A1 and A2 was to reduce the heat-leakage taking place in the cross-channel. In the block S' of plug A4, the diameter of the cross-channel is $\frac{1}{2}$ inch, and it is located with its axis a little above the axis of the block, which is otherwise similar to the blocks used with the other plugs. By thus enlarging the area of the cross-channel, the velocity of the steam while passing through it was made practically the same as in the secondary superheater in the oil-bath.

In plugs A1, A2 and A3, the lining in the tubes T and T' is of talc having a radial thickness of $\frac{3}{4}$ inch and an inside diameter of $1\frac{1}{2}$ inches. In plug A4, this lining is of poplox of about the same thickness, and a system of brass baffle plates separated by layers of copper gauze is located in each passage just above the soapstone block S. Copper baffles are also located in the chambers between blocks S and S'. These chambers, in plugs A2 and A3, contain a few layers of copper gauze, and in plug A1 are empty. No baffling devices are located in the talc-lined passages of tubes T and T' in any of the plugs A1, A2, A3.

2°. V1 and V2 refer to two radial-flow set-ups in the V-type of plug-case, illustrated in Fig. 3 and described in Section II, 2, *b*. V1 represents results with the plug case externally lagged with poplox and wholly submerged in the oil bath, V2 results with the plug-case directly immersed in the bath with no intervening lagging. The two other arrangements used with this type of plug case ((ii) and (iii), page 742) resulted in large heat-leaks, as might have been anticipated, and are of no interest.

3°. U1 and U2 refer to two radial flow set-ups in the U type of plug-case, described in Section II, 2, *c*. These set-ups are similar in all respects except that, with U1, a copper gauze strainer was used in the chamber M, Figs. 5 and 6, while with U2, an alundum strainer was used. (Only one strainer was used with plugs A1, A2, A3, A4, V1, V2, U1 and U2, there being no strainer corresponding to that shown in chamber m, Figs. 5 and 6).

4°. S refers to a radial-flow set-up of the straight-away type, described in Section II, 2, *d* and illustrated in Figs. 4, 5 and 6.

All the μ' vs. $\frac{1}{f}$ plots (Figs. 12 and 13) represent least-square adjustments of the observations. All the straight lines drawn on the $\frac{1}{\mu'}$ vs.

$\frac{1}{f}$ plots (Fig. 14) also represent least-square adjustments. The dotted curves on this diagram represent the corresponding least-square μ' vs. $\frac{1}{f}$ lines: where no dotted curve is drawn, the two loci are so close together as to be barely distinguishable. The numbers adjoined to each plotted point represent the number of runs involved, and, as a rule, also the weight attached to the point in making the least square adjustment.

The extrapolated value of μ (3°.182 C. cm.²/kgm.) as given by the set-up S is believed to be the true value of μ within $\frac{1}{2}\%$, for reasons which will appear in the following discussion. This value of μ is indicated on Figs. 15 and 16 by the slope of the straight dotted line passing through the origin. Throughout the following discussion, to avoid circumlocution, the value of μ with which these lines are associated will be spoken of as the true value of μ.

The actual mean temperatures of the various experiments range from 162°.6 C. to 168°.9. In reducing the observed μ's to the standard temperature of 165° C., the value -0.031 cm.²/kgm., taken from Fig. 7 of Davis' paper, 'On the Applicability of the Law of Corresponding States to the Joule-Thomson Effect in Water and Carbon Dioxide,'[8] was used for $d\mu/dT$. This value is undoubtedly sufficiently accurate for the purpose. A few measurements of μ for steam at 220° C. made by the writer verify the trend of Davis' curve. Moreover, the deviations of the temperatures at which the individual μ''s belonging to a given curve were experimentally determined, from the mean of these temperatures for this set of μ''s, are what are chiefly significant as regards error due to an erroneous value of $d\mu/dT$. This is because any error in the reduced values of these μ''s will be magnified in the extrapolation. In the case of every curve, these deviations were much smaller than the 2.4 to 3.9 degrees C. by which the temperatures of the extreme μ''s depart from 165° C.

In general, it is the high-side pressure rather than the mean pressure which was held approximately constant in the experimental work, so that, strictly, a reduction for pressure should also have been made before plotting the observations. No such reduction has been made. Its effect would certainly be almost negligible and would probably be to increase slightly the slopes of all the plotted straight lines of Figs. 12, 13 and 14, and thus also the μ's obtained by extrapolation.

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8 Davis, Proc. Am. Acad., **45**, 243–264 (1910).

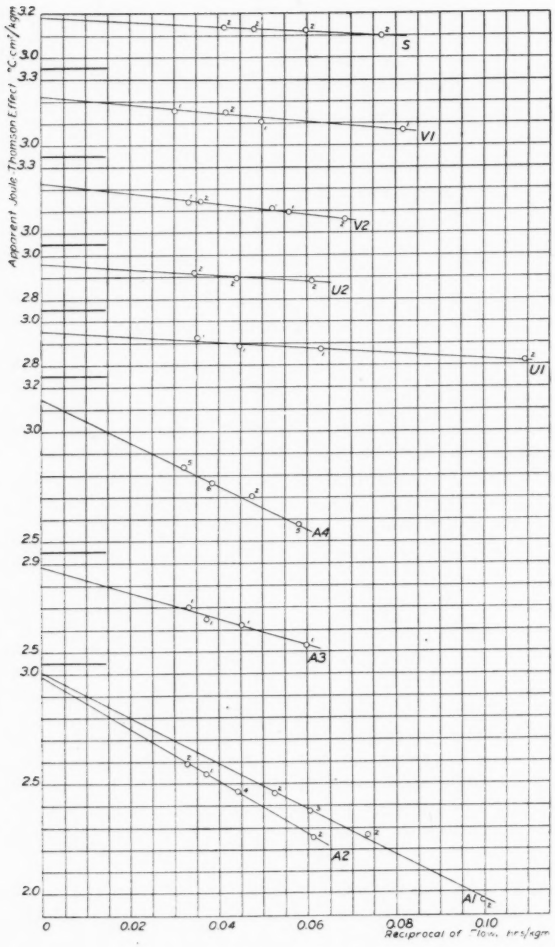

FIGURE 12. Apparent Joule-Thomson effect ($\mu' = \Delta T/\Delta p$) vs. reciprocal of flow. All μ''s are reduced to a mean temperature of 165° C. The average mean pressure, all values of μ', is 4.02 kgm./ cm.² abs. The average mean pressure for the runs with plug S is 3.86 kgm./cm.² abs. A1, A2, A3 and A4 refer to axial flow plugs, U1, U2, V1, V2, and S to radial flow plugs. The intercept of the line S on the axis of μ' is believed to be the true value of μ, within $\frac{1}{2}$%. The numbers adjoining the plotted points indicate the number of runs involved.

It was deemed best to omit it because no reliable data for calculating it are available. The question of the pressure coefficient of μ is briefly considered in a later section of this paper (Section IV, 2, *h*). The pressure given in the title to Fig. 12 (4.02 kgm./cm.^{2}) is the average mean pressure for the nine types of plug. The average mean pressure for the set-up S is also separately given.

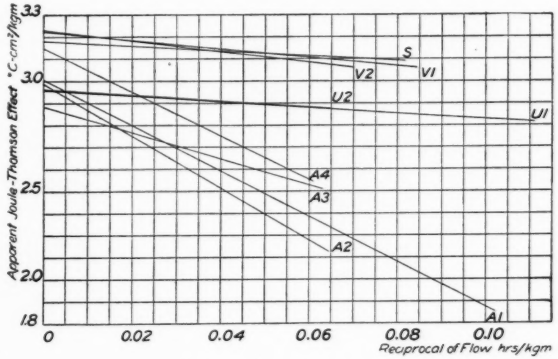

FIGURE 13. Curves of Fig. 12 shown with a common origin for comparison purposes.

b. Axial-flow Plugs A1, A2 and A3.

It will be noticed immediately (Fig. 14) that the plots of $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ for plug A2 shows very well marked curvature. The plots for plugs A1 and A3 are not shown, but are similarly curved. The plots for the other plugs (excepting possibly U1) do not present certain evidence of curvature which can be detected by the eye; but, with one exception (curve U1), in all cases in which the constants of the plot have been determined by the method of least squares, the residuals indicate that the μ' line represents the observed values of μ' with considerably more accuracy than the $\frac{1}{\mu'}$ line represents the observed values of $\frac{1}{\mu'}$. One concludes, therefore, that the heat-leak in unit time is not even approximately proportional to the temperature drop for the axial flow plugs, in which it is large, and that in general it

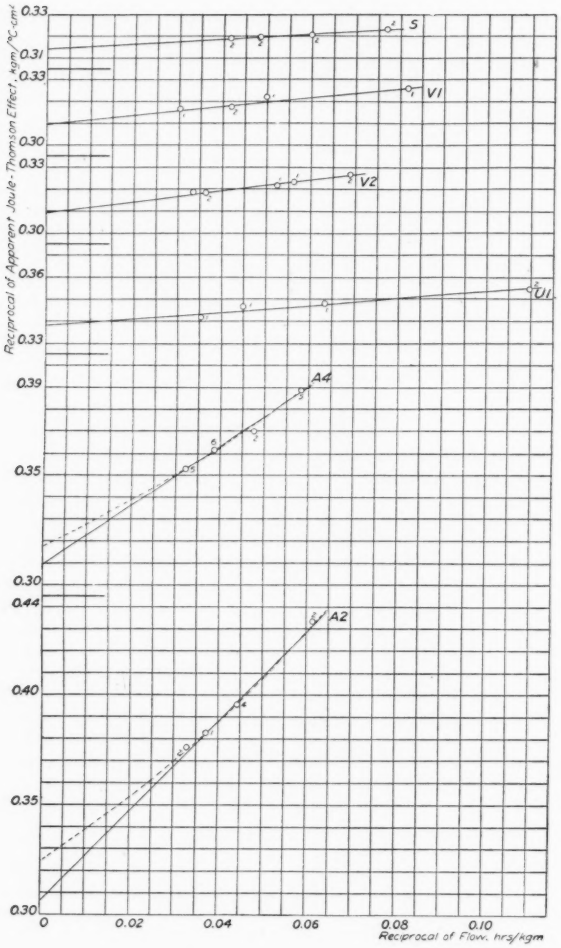

FIGURE 14. Reciprocal of apparent Joule-Thomson effect ($1/\mu' = \Delta p/\Delta T$) vs. reciprocal of flow. All $1/\mu'$'s reduced to a mean temperature of 165° C. The average mean pressure, all values of $1/\mu'$, is 4.02 kgm./cm.² abs. A2 and A4 refer to axial flow plugs, U1, V1, V2 and S to radial flow plugs. The numbers adjoining the plotted points indicate the number of runs involved.

varies more nearly as the pressure drop — is, in fact, in practically every case, proportional to pressure drop, within the limits of experimental error. Since the temperature drop increases more rapidly than the pressure drop in all cases, this means that the heat leak per unit time increases less rapidly than the temperature drop.

The very large leakage in plugs A1, A2 and A3 is partly due to the unusually happy combination for the promotion of heat-leak afforded by the long and rather constricted cross-channel used in these plugs. The comparatively small area of cross-section of the cross-channel, by forcing a high steam velocity and thus increasing the kinetic energy of the steam at the expense of its internal energy, depresses the temperature of the steam while it is in the cross-channel by an amount which, for high velocities, varies nearly as the inverse fourth power of the diameter of the cross-channel, for a given flow, and as the square of the flow for a fixed cross-section of the cross-channel. Since for these plugs the flow is approximately proportional to the square root of the pressure-drop, the temperature depression due to kinetic energy effect alone in the cross-channel will vary directly as the pressure drop. To this must be added the depression due to the throttling which has taken place in the high-side half of the plug, which is also proportional to pressure drop. Of course this total cross-channel depression of temperature is, in the steady state, reduced to some extent by the heat-leakage itself. It might be expected that the kinetic energy effect would predominate. This is not borne out by the facts, however; for plug A2, which permits more than 50 per cent. greater flow for a given pressure drop than plug A1 (see Table II) would produce a kinetic energy temperature depression more than twice as great as plug A1 for a fixed pressure drop, while the throttling depression would be the same for both. If then the kinetic energy effect were the dominant one, the heat leak per gram of steam at a fixed pressure drop might be expected to be greater for plug A2. The contrary is the case, as is shown by the ΔT vs. Δp plot. The heat-leak per gram of steam per unit pressure drop for a given flow is, however, greater for plug A2, as is indicated by the μ' vs. $\frac{1}{f}$ plot. This is because the pressure drop for a given flow is smaller for A2 than for A1.

The ΔT vs. Δp plots for these three plugs are all concave upward, though only slightly so; this means that the heat leak per gram of steam, which increases fairly rapidly with the pressure drop throughout the range of the experiments for all three plugs, increases a little

TABLE III.

Summary of plotted values of apparent Joule-Thomson effect and of its reciprocal, with other pertinent data.

All μ''s reduced to 165° C mean temperature. p_m signifies mean of high- and low-side pressures, f signifies steam flow. The last column indicates the number of runs involved for each plotted point.

Plug	p_m kgm/cm²	f kgm/hr.	$\frac{1}{f}$ hr./kgm	μ' °C cm²/kgm	$\frac{1}{\mu'}$ kgm/°C cm²	No. of runs
A1	4.30	10.00	0.10000	1.964	0.5091	2
	4.06	13.56	0.07373	2.266	0.4413	2
	4.00	16.49	0.06065	2.376	0.4208	3
	3.91	19.00	0.05262	2.414	0.4070	2
Mean p_m,	4.07					
A2	4.19	16.28	0.06144	2.256	0.4432	2
	4.06	22.54	0.04437	2.465	0.4057	4
	3.92	26.73	0.03742	2.545	0.3929	1
	3.83	30.31	0.03299	2.590	0.3861	2
Mean p_m,	4.00					
A3	4.27	16.68	0.05998	2.528	0.3956	1
	4.20	22.08	0.04531	2.620	0.3817	1
	3.88	26.82	0.03728	2.645	0.3781	1
	3.75	30.00	0.03333	2.699	0.3705	1
Mean p_m,	4.02					
A4	4.23	17.18	0.05821	2.575	0.3883	5
	4.20	21.02	0.04756	2.704	0.3699	2
	4.11	25.96	0.03852	2.765	0.3616	6
	4.06	31.18	0.03207	2.836	0.3527	5
Mean p_m,	4.15					
V1	4.20	12.24	0.08172	3.068	0.3259	1
	4.16	20.07	0.04983	3.106	0.3220	1
	4.07	23.92	0.04179	3.150	0.3174	2
	3.95	33.24	0.03008	3.159	0.3166	1
Mean p_m,	4.09					
V2	4.10	14.53	0.06878	3.062	0.3266	2
	3.97	18.46	0.05418	3.103	0.3223	2
	3.83	27.70	0.03610	3.144	0.3181	2
	3.78	30.04	0.03329	3.140	0.3185	1
Mean p_m,	3.92					

Plug	pm kgm/cm²	f kgm/hr.	$\frac{1}{f}$ hr./kgm	μ' °C cm²/kgm	$\frac{1}{\mu'}$ kgm/°C cm²	No. of runs
U1	4.24	9.12	0.10958	2.820	0.3546	2
	4.13	15.82	0.06320	2.873	0.3480	1
	4.06	22.29	0.04486	2.885	0.3466	1
	3.94	28.43	0.03517	2.926	0.3418	1
Mean p_m,	4.09					
U2	4.15	16.33	0.06122	2.882	0.3470	2
	4.03	22.66	0.04414	2.896	0.3453	2
	3.88	28.96	0.03454	2.920	0.3425	2
Mean p_m,	4.02					
S	4.13	13.01	0.07692	3.096	0.3230	2
	3.92	16.78	0.05982	3.120	0.3205	2
	3.76	20.78	0.04818	3.128	0.3197	2
	3.64	24.14	0.04148	3.136	0.3189	2
Mean p_m,	3.86					

Mean p_m, all plugs, 4.02 kgm./cm².

TABLE IV.

Characteristics of μ' vs. $\frac{1}{f}$ plots.

μ'_0 is the extrapolated value of the Joule-Thomson coefficient; μ'_2, the apparent value at the highest flow; μ'_1, the apparent value at the lowest flow. The quantities in the fifth column may be taken as representing the percentage part of the extrapolated coefficient which the extrapolation is depended upon to supply.

Plug	μ'_0	μ'_2	μ'_1	$\frac{100}{\mu'_0}(\mu'_0-\mu'_2)$	$\frac{100}{\mu'_0}(\mu'_0-\mu'_1)$	Geometrical Slope	Range of extrapolation / Range of observation
A1	3.007	2.465	1.974	18.0	34.4	0.517	1.11
A2	2.988	2.599	2.260	13.0	24.4	0.592	1.16
A3	2.885	2.688	2.529	6.8	12.3	0.297	1.25
A4	3.149	2.832	2.572	10.1	18.3	0.493	1.23
V1	3.224	3.167	3.065	1.8	4.9	0.096	0.58
V2	3.230	3.150	3.061	2.5	5.2	0.121	0.94
U1	2.956	2.912	2.877	1.5	2.7	0.064	0.47
U2	2.961	2.917	2.878	1.5	2.8	0.073	1.30
S	3.182	3.139	3.098	1.3	2.6	0.055	1.17

less rapidly as the pressure-drop is raised. The general trend of these plots as compared with that of the line representing the true value of μ

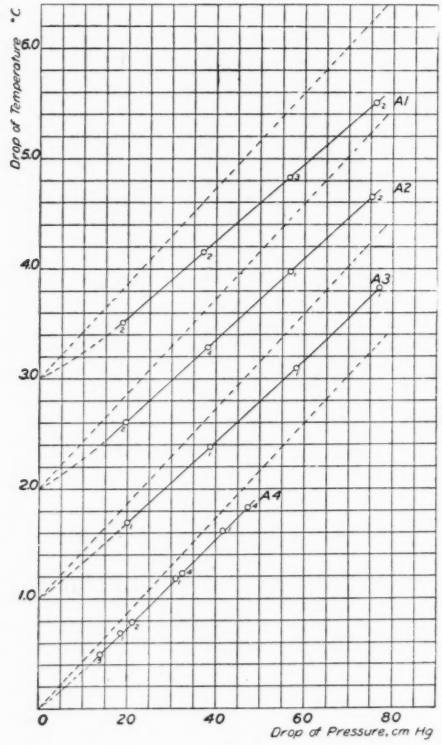

FIGURE 15. Temperature drop vs. pressure drop, axial flow plugs. The numbers adjoining the plotted points indicate the number of runs involved. The numbers along the axis of ordinates are for plug A4, the curves being spaced at intervals of 1° to avoid confusion. The straight dotted lines have slopes equal to the extrapolated μ for plug S.

illustrates, probably better than anything else, the general hopelessness of attempting to obtain reliable values of μ with such apparatus as this. The ΔT vs. Δp plot for plug A3 shows smaller heat-leakage

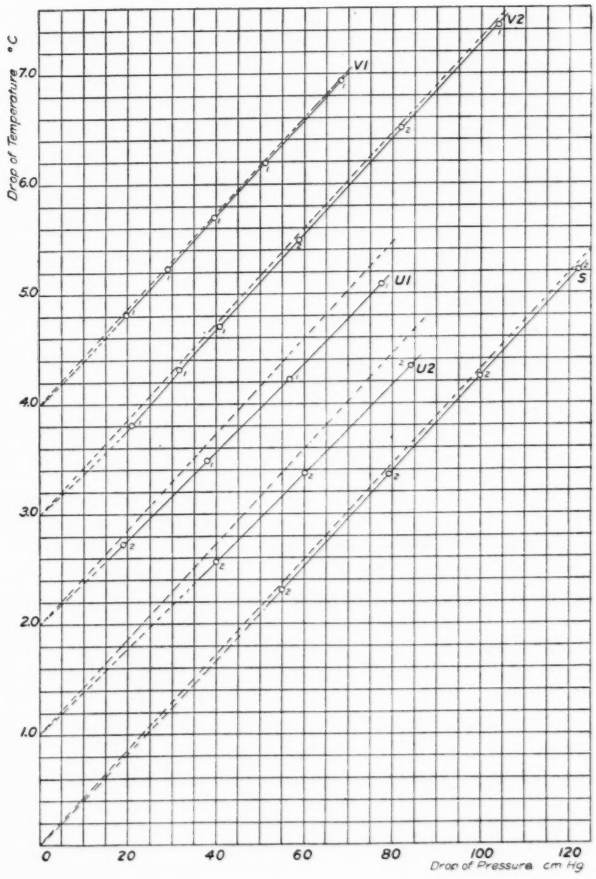

FIGURE 16. Temperature drop vs. pressure drop, radial flow plugs. The numbers adjoining the plotted points indicate the number of runs involved. The numbers along the axis of ordinates are for plug S, the curves being spaced at intervals of 1° to avoid confusion. The straight dotted lines have slopes equal to the extrapolated μ for plug S.

per gram of fluid than the others; this is due to the better insulation of the cross-channel in this plug.

The μ' vs. $\frac{1}{f}$ plots for these three plugs indicate clearly that the range of observation would have to be much greater than that actually covered by the experiments to justify any expectation of reliable results, and that, if the range were extended, these loci would necessarily be curved and hence that their extrapolation would be difficult or impossible. The extrapolated μ's from these plots are from 6 to 9 per cent. too small. It is probable that the small value of the extrapolated μ for A3 is due in part to an uncertainty regarding the pressure-drops for some of the runs involved in this plot, due to a difficulty with the zero correction to the manometer which has already been mentioned (page 752).

c. Axial flow plug A4.

The results obtained with the set-up A4, and represented by the lines so marked, are the best that have been secured with an axial flow plug and are probably as good as can be obtained with apparatus of the particular type used. The number of runs involved is excessive, amounting in all to seventeen, for a single value of μ which is about 1% too low. The very marked improvement, as regards heat-leak, over the axial flow plugs A1, A2, and A3, is due chiefly to the larger cross-channel, the kinetic energy depression of temperature being done away with. It is probable also that the improvement in thermal insulation effected by the use of poplox lagging in the vertical tubes T, T' of Fig. 1 is in part responsible for the reduction of heat-leak errors.

The ΔT vs. Δp plot probably shows to the best advantage the superiority of this plug to the other axial flow plugs. The heat leak per gram of steam is, generally speaking, less than half as great for a given pressure-drop as it is for plugs A2 and A3, and increases much less rapidly with increasing pressure drop than in these plugs. This is partly due to the larger flow per unit Δp in plug A4. The slope of the ΔT–Δp plot at its upper extremity approximates the true value of μ much more closely for plug A4 than for any of the other axial flow plugs.

All of the μ' vs. $\frac{1}{f}$ plots for the axial flow plugs have large slopes,

and it is therefore to be expected that the $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ plots will exhibit curvature. They all do, but the curvature is least distinct for plug A4, and, owing to the way in which the experimental errors distribute themselves, the curvature of the A4 plot might almost be considered accidental. It is interesting to note, in this connection, that if either of the μ' vs. $\frac{1}{f}$ or the $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ plots is straight, the other will be concave upward; and that a rectilinear extrapolation of a μ' vs. $\frac{1}{f}$ plot which is slightly concave upward will lead to too small a value of μ, while an extrapolation of a similar $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ plot will lead to too small a value of $\frac{1}{\mu'}$ i. e., to too large a value of μ. Hence if, in any given case, both plots seem sensibly straight, it might be expected that the true μ would lie somewhere between the two values obtained by these extrapolations. In the case of plug A4 the mean of these two extrapolated values is 3°.190 C. cm²/kgm, which is within 0°.008 C. cm.²/kgm. or $\frac{1}{4}$ of 1% of what has been taken as the true μ in this discussion.

d. Radial flow plugs V1 and V2. The regeneration effect.

The plotted points on the μ' vs. $\frac{1}{f}$ diagram for plug V2 (no lagging) and, possibly, also for plug V1 (external poplox lagging) are so situated as to admit the possibility of some real curvature in these plots, the curvature being negative (concavity downward). This curvature, reversed, is more evident in the $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ plots for these two plugs. It is probable, therefore, that rectilinear extrapolation of either plot for either plug is of rather doubtful reliability, in spite of the comparatively small slope of the lines concerned. However, the deviations of the plotted points from the least square straight lines are not too large to be ascribable to accidental errors, at least in the case of the μ' vs. $\frac{1}{f}$ plots, and even if they were, and if some better representation could be devised, the plots in their present form are useful for comparison purposes. They are therefore shown.

Both of the μ' vs. $\frac{1}{f}$ plots, on this rectilinear representation, extrapolate to values of μ which exceed the true value by about 1 per cent., the result for the unlagged plug (V2) being slightly greater than that for the other. (Both extrapolated values would of course be too large if there is genuine negative curvature in the loci.) The slope is greater for the unlagged plug than for the other.

The ΔT vs. Δp plots exhibit definite characteristics unlike those of any other plug. Both are concave downward within the region of observation, the heat-leak per gram of fluid decreasing with increasing pressure drop, with a well-marked tendency to approach a limiting value which is nearly zero for V1, the lagged plug, and which, for V2, is small and of about the same magnitude as the corresponding leak for plug S. Both plots would show points of inflexion if observations had been taken at smaller pressure drops, since ΔT and Δp vanish together; this is indicated by the dotted portions of the plots.

This peculiarity is believed to be due to the operation of the regeneration effect discussed under II, 2, *a*. As is there pointed out, this effect produces an outward heat-leakage from the steam between the thermometers, the effect of which is to oppose the normal inward heat-leakage due to conduction. The external lagging used with plug V1 assists the operation of this regenerative action, since it interferes with the flow of heat from the bath to those portions of the passing fluid upon which the regenerative action takes place. It is on this account that the net inward heat-leak is larger for the unlagged than for the lagged plug. It seems probable that the total outward heat leak due to regeneration would increase at about the same rate as the pressure drop, and since the flow varies approximately as the $\frac{3}{4}$ power of the pressure drop, the outward heat leak per gram of fluid due to regeneration would increase with the pressure drop; moreover, since the inward leak per gram of fluid due to conduction, etc., increases with pressure drop in the case of every plug in which this is the only leak occurring (including plug S), it is evident that the net result might be to produce a heat-leak per gram which would tend toward some limiting value such as the experimental results actually indicate. It is hardly possible to be more definite than this regarding the effect which might reasonably be anticipated from the joint operation of the several agents at work, because of the complicated nature of the phenomena concerned.

The μ' vs. $\frac{1}{f}$ plots also point, though less clearly, to the operation of

regenerative action. The greater slope of the V2 (unlagged plug) line is due to the inhibition of the corrective action of the regenerative leakage by the flow of heat from the bath. While this line extrapolates to a larger value of μ than the other, the difference is slight and is of much less significance than the definite difference of slopes, in view of the comparatively large deviations of the plotted points from the representative straight lines, (or possibly, the genuine curvature of both plots).

The important conclusion from the work with these set-ups is that the effect of regeneration is not great. This inference is drawn from the observation that only relatively insignificant changes in heat leak were produced by making changes in lagging which must have had a very important effect on regeneration. This is the principal justification for the belief that regenerative action is practically negligible in plug S, in which heavy *internal* lagging is provided.

It is to be noticed that the slopes of the upper portions of both ΔT–Δp plots are practically identical with what has been called the true μ.

e. Radial Flow Plug S.

This represents the final form of the apparatus. The μ' vs. $\frac{1}{f}$ and $\frac{1}{\mu'}$ vs. $\frac{1}{f}$ plots both have smaller slopes than with any other plug, and neither exhibits any evidence of curvature. The deviations of the plotted points from the representative straight lines are small, and the value of μ as derived from the $\frac{1}{\mu'}$ plot exceeds its value as derived from the other by only 0°.002 cm.²/kgm., or less than $\frac{1}{15}$ of 1%. All the points of the ΔT vs. Δp plot lie on a straight line, the slope of which is very slightly less than the true μ. The heat-leak per gram of steam, which is probably due chiefly to conduction down the walls of the plug, thus still increases slightly with the pressure drop, but the increase is so slight that it would disappear or become a decrease if the value of the 'true μ' were changed by only a small amount. There is no evidence like that which, in the case of plugs V1 and V2, was interpreted as pointing to regenerative heat-leak.

The reasons for believing that the value of μ (3.182), given by the

rectilinear extrapolation of the μ' vs. $\frac{1}{f}$ plot for this plug, is the true value within $\frac{1}{2}$ per cent., may be summarized as follows:

1°. The results show that the set-up was to a large degree successful in avoiding the sources of error it was designed to avoid. The ordinary conductive heat leak is very small and, as already noted, there is no evidence of regenerative leak. The actual percentage of the true μ which the extrapolation is depended upon to supply is only 1.3 per cent.

2°. It is very probable that the true value of μ lies somewhere between the values given by plugs V1 or V2, which are too high because of regeneration (or possibly because the μ' vs. $\frac{1}{f}$ plots for these plugs are really concave downward instead of rectilinear — and, if this is so, it is doubtless itself due to regeneration), and the value as given by the μ' extrapolation for plug A4, which is too low. (Plug A4 is the best of a series of axial flow plugs, all of the earlier of which give extrapolated μ's which are certainly much too small.) The extrapolated μ's for plugs V1 and V2 are respectively 3.224 and 3.230 while that for plug A4 is 3.149. Also, it has already been noted that the mean μ as determined from the two A4 plots differs by only 0.008 — about $\frac{1}{4}$ per cent.— from the extrapolated μ of plug S.

3°. Where the slopes of the ΔT vs. Δp plots reach, or approximately reach, a limit at or below the highest pressure drops, these limiting slopes may be regarded as representing approximately the true value of μ. If the calculations of slope are made by subtracting one observed temperature drop from another and dividing the result by the difference of the corresponding pressure drops, it is found that none of the set-ups except V1, V2 and S show evidence of a limiting value of the slope such as has just been mentioned. In the case of these three plugs, the values of the slope, calculated as described from the two highest pressure drops, are 3.20, 3.18, and 3.18 degrees C. per kgm. per cm.^{2}, respectively. These values would probably be slightly changed if the smoothed curves, instead of observed points, were used as a basis for the calculation; but in either case, the method of calculation would necessarily be somewhat inaccurate, and the results are offered only as confirmatory of the result (3°.182 C. cm.^{2}/kgm.) obtained by extrapolating the μ' vs. $\frac{1}{f}$ plot for plug S.

4°. The μ' vs. $\frac{1}{f}$ plot for plug S has the least slope of all such plots,

and the points plotted to determine it show that the accidental errors were smaller for plug S than for any other plug (see Table IV and Fig. 12). There are several other plugs for which the ratio of the range of extrapolation to the range of observation is less than for plug S, but, as an inspection of Fig. 13 will show, the inferiority of plug S in this respect in a comparison with any of these plugs is outweighed by its marked superiority in one or both of the other two respects.

In the case of plug S, as has already been stated, the difference between the value of μ' at the highest flow and its value on the axis of $\frac{1}{f}$ is only 1.3 per cent. of the latter. The extrapolation must therefore be supposed to be so uncertain that the part of the extrapolated μ which depends upon it is in error by 40 per cent. of itself, if one is to believe that an error as great as 0.5 per cent. is involved from this cause alone. A rough idea regarding the probability of a percentage extrapolation error of this magnitude in the curve of plug S may be obtained by considering the extrapolation errors of other plugs.

If it is assumed that the constant A in equation (2a) is zero, all of the deviation of any extrapolated μ from the true μ must be assigned to extrapolation error, in the sense in which this expression has heretofore been used. Now, whether the precision claimed for the μ given by plug S is allowed or not, we may certainly suppose that 3.18 represents the true value of μ within, say, 2 per cent. — that is, that μ certainly lies somewhere between 3.15 and 3.21. For any plug, the extrapolation error will then lie between $3.15 - \mu'_0$ and $3.21 - \mu'_0$, where μ'_0 is the extrapolated μ. The percentage error of extrapolation is 100 times the ratio of the difference between the true μ and μ'_0 to the difference between μ'_0 and μ'_2, where μ'_2 is the apparent value of the coefficient observed at the highest flow. Hence the percentage error of extrapolation lies between $100\ (3.15-\mu'_0)/(\mu'_0-\mu'_2)$ and $100\ (3.21-\mu'_0)/(\mu'_0-\mu'_2)$. The values of these limits may be calculated from the data given in Table IV; they are found to be, for plug A1, 20 and 52 per cent.; for plug A2, 34 and 64 per cent.; for plug A3, 128 and 192 per cent.; for plug U1, 320 and 560 per cent.; for plug U2, 400 and 700 per cent. For the other plugs, μ'_0 lies too close to 3.18 for the calculations to have much significance.

It is thus seen that, for an error as great as $\frac{1}{2}$ per cent. to be present in the μ obtained by extrapolating the plot of plug S, we must suppose the percentage extrapolation error with this plug to be of the same order of magnitude as with plugs A1 and A2 and from a third to a fifth as large as with plug A3. One is justified in believing that the

error is considerably smaller than this. It is true that it is necessary to suppose the percentage extrapolation error with plug S to be only $\frac{1}{8}$ to $\frac{1}{18}$ of this error with plugs U1 or U2, in order thereby to account for an error of $\frac{1}{2}$ per cent. in what we have called the true μ. But it is extremely doubtful that the plots for plugs U1 and U2 can be regarded as straight within experimental error, and therefore difficult to say by how much the extrapolation error with these plugs might reasonably be expected to exceed that with plug S, for which the rectilinearity of the plot is more secure than for any other plug. Moreover, the large values of the percentage extrapolation error for plugs U1 and U2 are due to the small slopes of the plots and would be materially changed by comparatively small changes in these slopes.

5°. Nothing whatever as to the legitimacy of the assumption that the A of equation (2a) is zero can be inferred either from the straightness of a given plot or the smallness of its slope, and this assumption has exactly the same validity in the case of plug S as it has in the case of any other plug for which the value of the exponent r of equation (4) is the same. But certainly it is not easy to see how or why a heat leak per unit time varying as the 1.75 power of the pressure drop — practically as the 1.75 power of the temperature drop — and of any appreciable magnitude, could be present.

6°. The effect of failure to correct for the pressure coefficient of μ probably is to make the extrapolated μ too small, but not, it is believed, by an amount as great as 0.3 per cent. (See Section IV, 2, *h*.)

7°. The precision of the temperature measurements is at least as good as $\frac{1}{5}$ per cent., and that of the pressure measurements $\frac{1}{10}$ per cent., or even better at the higher flows.

f. Measurements of μC_p and the calculation of μ from them.

To measure directly the value of the product μC_p, it is necessary to supply energy to the fluid during its passage through the plug, of an amount sufficient to prevent the ordinary Joule-Thomson drop of temperature. In the case of such axial flow plugs as have been used in this work, this energy is supplied electrically by means of the heating coil shown in Fig. 1. If the supply of energy is just sufficient to maintain the low-side temperature equal to the high side temperature, it may be assumed, in the absence of such effects as varying kinetic energy in different parts of the plug, that the temperature of the steam is that of the oil-bath during the whole of its passage through

the plug, and hence that there is no heat-leakage. Under these circumstances, the energy supplied per unit mass of fluid is equal to $\mu C_p \cdot \Delta p$, in which μ is evaluated at the mean of the high and low side pressures, and at the mean of the high side temperature and of what would be the low side temperature if the experiment were adiabatic, and C_p is evaluated at the mean of these temperatures and at the low side pressure. The product μC_p is in several respects of more importance than μ alone, and obviously a direct measurement of μC_p has advantages over its determination by separate measurements of μ and of C_p, particularly as it is the manner of variation of μC_p with pressure and temperature which it is useful to know.

In the present work, eleven measurements of μC_p have been made, all at approximately the same pressure and temperature at which the μ-measurements were made. Two were made with plug A2, four with plug A3, and five with plug A4. The measurements, when combined with the low values of μ obtained with the earlier apparatus, led to values of C_p which were so much in excess of the reliable results obtained by Knoblauch [9] and by Knoblauch and Mollier,[10] that the inference seemed unavoidable that some large source of outward heat-leak must have been operative. In addition to this, the measurements were discordant to a degree indicating accidental errors of much greater magnitude than occur in the adiabatic experiments. The results on μ obtained with the later apparatus bring the mean of these determinations of μC_p into much closer agreement with Knoblauch and Mollier's values of C_p, however. These μC_p measurements thus afford another check on the accuracy of the value of μ obtained in the experiments already discussed. They are not offered as representing dependable values of μC_p but merely as having a certain bearing on the legitimacy of the claim which has been made in respect to the reliability of μ as determined by the S-type of radial flow plug.

If each of the several measurements of μC_p is divided by the proper value of C_p, taken for the conditions of the experiment from table 4 of the paper by Knoblauch and Mollier above referred to, there result eleven values of μ. After these are reduced to 165° C., their mean is 3°.26 C. cm²/kgm., with an average deviation for the entire eleven of 0°.09 C. cm.²/kgm. If one divides this deviation by the square

9 Knoblauch and Jakob, Mitteilungen über Forschungsarbeiten des Vereines deutscher Ingenieure, **35**, 109 (1906).

10 Knoblauch and Mollier, ZS. des Vereines d. Ing., 1911, p. 655.

root of the number of determinations, one finds $0°.02_7$ as the deviation of the mean. This is little more than nominal, for it is highly improbable that the large errors involved were all accidental. Accepting it, however, the difference between the value of μ as derived from these measurements and the value obtained from the experiments with the S-type of plug is about three times the deviation of the mean of the results of the μC_p measurements — that is, about 2.5 per cent. of μ itself. There does not appear to be any reason for suspecting that the experimental work of Knoblauch and Mollier was such as to involve a constant error in either direction. Their experimental errors, as indicated by the values of C_p plotted on Fig. 5 of their paper, were of the order of about one per cent. in the region of the plane here involved. It happens that their C_p curve for 4 kgm./cm.^{2}, the position of which largely determines the values of C_p used in the calculations of μ, is for some reason drawn by them along the lower edge of their band of experimental points in the temperature region within which the μC_p measurements were made. If the curve had been here so located as to pass through the mean position of their experimental points, about one-third of the discrepancy of 2.5 per cent. noted above would disappear.

As has been already stated, the μC_p experiments were dropped rather early in the present research in order to settle the heat-leak question in the adiabatic work, and there is not sufficient experimental evidence to yield any conclusion of consequence regarding a constant error of either sign, or regarding the cause of the large experimental errors, in them. With plugs A2 and A3, the effect of high steam velocity in the cross-channel would be to make the measured values too small. This effect ought to be absent from the results obtained with plug A4, which are, in fact, larger than with the other plugs. Variations in the provisions for mixing the steam produced no consistent results. It was suspected at first that an outward heat-leak might have been due to the fact that, in order to get heat from the heating-coil into the steam, it had been necessary to make the temperature of the coil considerably higher than that of the oil bath. Measurements of the resistance of the manganin coil used with plugs A2 and A3 seemed to indicate this, but the indication was not reliable, owing to the small temperature coefficient of manganin. The matter was more thoroughly tested in plug A4 by employing a heating coil of stranded invar. This alloy has a high specific resistance and, for an alloy, a high temperature coefficient.* The resistance of the coil, at

* Mean coefficient, 30° to 170° C., 0.00145 per °C.

constant temperature, decreased slightly during the course of the experimental work, doubtless because of an aging effect, recovering a portion of the decrease during idle periods. An increase in the energy input at constant flow resulted in a decrease of the measured resistance in five cases and in an increase in three. The evidence is thus against the existence of a heat-leak of the kind suspected; it is probable that the variations of resistance actually noted were due to other causes than variations of the temperature of the coil.

g. The wet-steam question. Radial flow plugs U1 and U2.

It is improbable that perfectly dry superheated steam is ever secured in practice, at least at temperatures within thirty or forty degrees Centigrade of saturation. Particles of liquid are always brought away, suspended in the vapor, during vaporization. These particles, spherical in shape, are able to persist in the liquid form after the application of superheat because of the excess pressure on the interior of the drop produced by the surface tension. The effect of evaporation from the surface of the drop is a reaction against a continuation of the process, because of the increase in the curvature of the surface. Further, if, in some manner, steam free from moisture were actually obtained, it would be very difficult to demonstrate the fact.

As far as calorimetric effects are concerned, the presence of a small amount of moisture in superheated steam is not objectionable, provided the quantity present does not change as a result of the experimental processes. For example, if the steam contains 0.1 per cent of water, any temperature change undergone by the fluid will differ by only about 0.1 per cent. from what it would be if no water were present at all, *provided* the water-content is not changed; but if, for each degree of temperature change, as little as 1 per cent. of the 0.1 per cent. water-content is vaporized, the effect on the temperature change will amount to 1 per cent. (These statements suppose, as is roughly the case, that the latent heat of vaporization and the specific heats of water and steam are in the proportion 1000:2:1.)

What an experimenter should aim at, therefore, is to reduce the amount of moisture to as low a figure as possible and to endeavor to have the moisture which is unavoidably present in a form such that no change in its amount is likely to occur during passage through the calorimetric apparatus. After taking all the precautions which seem necessary to secure these conditions, they should be doubled. If no noticeable changes result, it may be regarded as reasonably certain

that the effect of moisture is negligible — not, however, that moisture is necessarily absent.

The presence of varying amounts of moisture in supposedly dry steam has been the occasion of so considerable an error in the results of at least one experimenter that it was deemed advisable, in the present work, to provide with some care against the entrance of error from this source. The precautions taken have already been described. They consisted in attention to the design of the primary superheater, in operating the secondary superheater as a cooler, and in providing strainers. The object of strainers is in part to get rid of moisture by the operation of throttling, on the principle embodied in the ordinary throttling calorimeter used for determining the quality of moist steam, and in part to break up particles of moisture into small drops, from which further evaporation is less likely to occur. The only actual tests on the efficacy of these precautions are given in the results obtained with plugs U1 and U2. The arrangements of the apparatus in these two cases differed only in that a copper gauze strainer was used with U1 and an alundum strainer with U2. A glance at the curves of Figs. 12 and 13 will show that this test indicates no appreciable wet steam effect. In fact, such discrepancy as appears is in the wrong direction for explanation on the supposition that wet steam was the cause of it, since the alundum strainer was far more effective than the gauze strainer.

The low values of μ obtained by extrapolation with the two U-type plugs are mainly due to heat-leakage into the annular space surrounding the radial flow plug. The steam is here spread out into a rather thin layer not effectively insulated from the bath, and as its temperature has been depressed, before it reaches this space, by throttling in the mixing chambers at the two ends of the cross channel, the opportunity for heat-leakage is excellent. The need of locating the high side thermometer in close proximity to the radial flow plug is well illustrated by these results. Although the set-up is similar to that used with the axial flow plug A4, the results do not form a fair basis of comparison between the two types of plug, since the chief advantage of the radial flow plug is sacrificed by the arrangement.

In both of the U-type plots of μ' vs. $\frac{1}{f}$, there is evidence of curvature, which, if actual, would lead to larger μ's than are given by the straight line extrapolation. Whether this curvature is real or not has no particular bearing on the evidence of the plots as to the question of wet steam, and the plots have practically no other value than this.

h. Existence of a pressure coefficient of μ.

In 1909, Davis [11] made a very complete study of all experimental results on the Joule-Thomson effect in steam which had at that time been published, and of some which had not been published. His purpose was to test the applicability of the law of corresponding states to the Joule-Thomson effect in steam and in CO_2. He states that a careful scrutiny of all the observations with which his paper concerns itself failed to reveal any systematic variation of μ with pressure, and concludes that if such a variation exists at all, it is within the limits of error of the experimental work. These limits are not narrow, but the wide range of pressure covered by the four experimenters of whose results Davis made use certainly justifies the inference that the pressure-coefficient of μ must be small to have escaped detection.

In the present research, a few experiments have been made at mean pressures of 83.0 lbs./in.^{2} (5.84 kgm./cm.^{2}) and 32.7 lbs./in.^{2}(2.30 kgm./cm.^{2}), these pressures being such that the mean pressure of the other experiments is approximately their arithmetic mean. The experiments referred to were made with the V1 plug, without interchange of thermometers. The μ' vs. $\frac{1}{f}$ plots are shown in Fig. 17, together with the curve for plug V1 already shown in Fig. 12. Owing to unusually large accidental errors (or possibly, genuine curvature of the plot) in the case of the highest pressure, and to the fact that the flow through the plug was comparatively small for the lowest, the rectilinear extrapolation of the μ' vs. $\frac{1}{f}$ plot is unreliable in both cases. Moreover, the presence of the regeneration effect in this plug diminishes one's confidence in employing the results to arrive at a value of the pressure coefficient of μ, particularly as this effect seems more prominent in the 5.84 kgm./cm.^{2} runs than in the others. There is also the possibility of wet steam in the 5.84 kgm./cm.^{2} runs, which were fairly close to the saturation line. A rough estimate of the magnitude of $(\partial\mu/\partial p)_t$ can, however, be obtained by considering those parts of the three curves on the μ' vs. $\frac{1}{f}$ diagram which overlap, ascrib-

11 Proc. Am. Acad., **45**, 243–264 (1910).

ing the difference in the average apparent Joule-Thomson coefficients over identical ranges of flow to the effect of pressure. The three values of $(\partial\mu/\partial p)_t$ obtained in this way are:

+0.036	°C. cm.⁴/kgm.²	for the	overlapping of the	5.84 and the 4.09 kgm./cm.² lines
+0.028	"	"	"	of the 4.09 and the 2.30 kgm./cm.² lines
+0.024	"	"	"	of the 5.84 and the 2.30 kgm./cm.² lines

Taking the mean of the first two and the mean of this mean with the third, there results + 0.028 °C. cm.⁴/kgm.² as the value of $(\partial\mu/\partial p)_t$. This is of course only a very rough approximation, and is probably an upper limit.

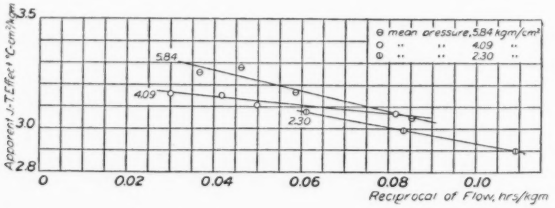

FIGURE 17. μ' vs. $\frac{1}{f}$ plots for three sets of runs at three different mean pressures, plug V1. Mean temperature, 165° C. in each case. Numbers attached to lines indicate average mean pressure, kgm./cm.²

In a paper on the properties of saturated and superheated ammonia vapor, Goodenough and Mosher [12] have re-examined the data discussed by Davis, in an effort to verify an empirical equation of state for ammonia vapor by comparing reduced values of μ as calculated from it with the reduced observed values used in Davis' paper. For this purpose they recomputed Davis' reduced results, using later critical data for water than Davis employed. In this way they were able to show that their calculated μ curves for reduced pressures corresponding to ammonia pressures of 4, 5, 7 and 9 atmospheres agreed well with the observed results of which Davis made use, when these results were grouped in such a way that the mean reduced pressure for each group was approximately the same as that of the curve with which comparison was made. Inferior agreement was obtained in the case

12 Goodenough and Mosher, Bulletin No. 66, Univ. of Illinois Engineering Experiment Station, January 27, 1913.

of the curves corresponding to the 2, 3 and 10 atmosphere ammonia curves, of a sort indicating that the Joule-Thomson coefficient does not vary with the pressure so rapidly as their equation demands.

The reduced mean coordinates of the runs with the set-up S are 0.0172 for the pressure and 0.676 for the temperature. The corresponding pressure and temperature for ammonia are respectively 29.05 lbs./in.^{2} abs. and 495° F. abs. On differentiating equation (p) of Goodenough and Mosher's paper with respect to the pressure at constant temperature, and inserting the above values of pressure and temperature, the values of the various constants as given by the authors and their values of C_p and its pressure-derivative as given by their equation (9), one finds the value −0°.0832 F. in.^{4}/lb.^{2} for $(\partial\mu/\partial p)_t$ for ammonia in the state specified. Multiplication of this by $(1690)^2/(460 + 273)$, which is the ratio of the square of the critical pressure of ammonia in lbs./in.^{2} to its critical temperature in °F. abs. gives −11.96 as the value of the derivative in question in reduced units. This value may be checked by reference to Figs. 9a and 9b of Goodenough and Mosher's paper. (Figs. 9a to 9e inclusive, except the lower part of Fig. 9c, of this paper, are all incorrectly labelled; 9a and 9b are for reduced pressures of 0.0174 and 0.0261 respectively, instead of 0.0870 and 0.0783. A first glance at this set of figures gives the impression that μ increases with increasing pressure at constant temperature, which is in contradiction of the equation (p) of the paper.) Finally if −11.96 be multiplied by $648/(225.0)^2$, we find −0.152 °C. cm.^{4}/kgm.^{2} for $(\partial\mu/\partial p)_t$ for steam under the conditions of the runs of set-up S, by this somewhat round-about invocation of the law of corresponding states.

There is no doubt that this result is far too large as regards its magnitude, and possibly incorrect in sign. Its only justification is that at other pressures the throttling experiments used by Davis verify Goodenough and Mosher's ammonia equation. The actual observations in the vicinity of the pressure we are here concerned with are not in agreement with this equation. If the experiments made by Dodge, which exhibit much larger accidental errors than those of the other observers, are neglected, it may fairly be doubted whether even the negative sign of the pressure derivative of μ is verified. At all events, the accidental errors of the experimental work are so large compared with the effect sought that any quantitative estimate would be hazardous, and none will be here attempted. Of course the very good verification obtained by Goodenough and Mosher at higher pressures raises the presumption that at the pressure of the

present writer's experiments, the pressure-coefficient of μ would still be negative, in spite of the contrary evidence yielded by the work which has been described. Enough weight is attached to this evidence, however, to discount to some extent the deductions from the ammonia equation, of which experimental verification at the pressure in question is lacking, and, in view of the conflict of evidence as to sign, to regard the pressure-corrections to the writer's observed μ's as negligible within the accuracy claimed for the μ of set-up S.

Goodenough [13] has proposed the empirical equation

$$v = c + \frac{BT}{p} - (1 + 3ap^{\frac{1}{3}}) \frac{m}{T^n} \tag{5}$$

for superheated steam. He finds that this equation is in excellent agreement with the specific volume determinations of Knoblauch, Linde and Klebe.[14] By means of well-known thermodynamic relations, the equations

$$C_p = \varphi(T) + \frac{Amn(n+1)}{T^{n+1}} p(1 + 2a p^{\frac{1}{3}}) \text{ and} \tag{6}$$

$$\mu = \frac{A}{C_p}\left[\frac{m(n+1)(1+3ap^{\frac{1}{3}})}{T^n} - c\right] \tag{7}$$

are deduced from it. By adopting for the arbitrary function $\varphi(T)$, in (6), the form

$$\varphi(T) = \alpha + \beta T + \frac{\gamma}{T^2} \tag{8}$$

Goodenough further secures good agreement with the C_p determinations of Knoblauch and Mollier [15] and other experimenters. If we calculate the value of $(\partial\mu/\partial p)_T$ from (7), using the proper values of the several constants,* and taking $p = 4.0$ kgm./cm.^{2}, $T = 165°$ C. ordinary $= 438°$ C. abs., we find $(\partial\mu/\partial p)_T = +0.014_7$ °C. cm.^{4}/kgm.^{2}, which is of the same sign as the roughly approximate value derived from the experiments with plug V1, though only about half as great.

13 Properties of Steam and Ammonia, Wiley and Sons, 1915, p. 6.
14 Mitt. über Forschungsarbeiten des Ver. d. Ing., **21**, 33–55 (1905).
15 Op. cit.

* If p is in kgm./cm.^{2}, v in m.^{3}/kgm., T in °C. abs., C_p in kgm. cals./kgm. °C., μ in °C. cm.^{2}/kgm., then $\log_{10}\beta = 3.67213$, $\log_{10} m = 8.59929$, $3a = 0.19339$, $n = 4$, $A = \frac{1}{6.04256}$, $\alpha = 0.320$, $\beta = 0.0002268$, $\gamma = 7371$. c is not strictly constant, being the specific volume of the liquid at the pressure under consideration. It has the value 0.001083 m.^{3}/kgm. in the present calculation.

In Goodenough and Mosher's ammonia equation, the constant a is zero and the constant c is negative. This accounts for the difference as to sign between the value of $(\partial\mu/\partial p)_T$ just calculated and that derived from the ammonia equation.

If the pressure coefficient of μ is actually negative, the right hand ends of all μ' vs. $\frac{1}{f}$ plots should be tilted upward, decreasing slightly the heat-leak effect. If it is positive, as the writer believes and as Goodenough's steam equation indicates, the extrapolated μ's will all be slightly increased. In the case of the plot of μ' against $\frac{1}{f}$ for plug S, for which the effect would be greatest, a reduction of all μ''s to 3.87 kgm./cm.^{2} on the assumption that $(\partial\mu/\partial p)_T = +0.028$ °C. cm.^{4}/kgm^2 raises the four plotted points by 0.006, 0.003, −0.002 and −0.008 °C. cm.^{2}/kgm. respectively, counting from the one for the smallest value of $1/f$ outward in order, and the extrapolated value of μ is increased from 3.182 to 3.200 — about 0.56 per cent. It is doubtful whether the actual correction is as great as half of this.

i. Comparison with the results of other experimenters.

Only three accounts of Joule-Thomson experiments by other investigators on superheated steam over ranges of temperature which include 165° C. have been published. These experiments were conducted by Peake,[16] Grindley [17] and Griessmann.[18] All were what engineers usually call 'throttling' or 'wire-drawing' experiments,— that is, experiments in which steam known or assumed to be dry and saturated is throttled to various lower pressures under conditions of negligible loss of energy during the throttling process. The curve which passes through a set of points representing the states of the fluid on the low side of the plug for a number of different pressure-drops from the same original high side pressure, is called a 'throttling curve,' and is a curve of constant total heat, if the throttling has been performed without gain or loss of energy. In all of the experiments of Grindley and in some of those of Peake, the steam on the high side was not devoid of moisture; hence, in these experiments, the state

16 Proc. Roy. Soc., **76 A**, 185–205 (1905).
17 Phil. Trans. **194A**, 1–36 (1900).
18 ZS. des Vereines d. Ing., **47**, 1852–1857 and 1880–1884 (1903).

point of the high-side steam does not lie on the throttling curve. However, in these as well as in experiments in which the high side steam was really dry, the slope of the tangent to any throttling curve plotted in the (p, T) plane is the value of μ at the point in question, provided, of course, that the throttling has been adiabatic.

Since the experiments in question were primarily undertaken for the purpose of determining sets of throttling curves rather than the slopes of these curves, none of the experimenters used differential apparatus in either the drop of pressure or the drop of temperature measurement. Both drops were usually much larger than any in the work of the present writer. Thermometers were not read more closely than 0°.1 C. and this place was, as a rule, uncertain. It is therefore to be expected that large accidental errors will appear in values of μ calculated from the work of these three investigators by taking differences between successive low-side pressure and temperature readings obtained with the same high side conditions. Such calculations have been very carefully carried through by Davis in a paper already referred to [19] and the results are embodied in tables I, II and III and in the figure 6, of that paper. Davis also exhibits in figure 7 of the same paper a curve representing the Joule-Thomson effect for steam as a function of the temperature. This curve is based, not only on the work of Peake, Grindley and Griessmann, but also on steam experiments at other temperatures by Dodge and on a number of experiments on carbon dioxide, it being assumed, with regard to the last-named, that the law of corresponding states holds for the Joule-Thomson effect in carbon dioxide and water. This curve gives about 3°.11 C. cm.²/kgm. as the value of μ for steam at 165° C.— about 2.2 per cent. lower than the value (3.182) obtained by the present writer with plug S.

The influence of the carbon-dioxide points is probably scarcely felt at the temperature in question, as a glance at Davis' figure 6 will show (165° C. = 0.687 reduced, using 365° C. for the critical temperature of water, as Davis did). However, to avoid any possible effect of this sort, and also to avoid an effect due to the manner in which Davis grouped the results of the throttling experiments in obtaining the coordinates of the points of his figure 6, the values of all Joule-Thomson coefficients given in Davis' tables I, II and III for temperatures lying within the interval 0.671 to 0.703 reduced (corresponding to 155° to 175° C. for steam) have been considered directly. There

19 Proc. Am. Acad., **45**, 243–264 (1910).

are thirty-five of these in all — eight of Grindley's, thirteen of Griessmann's and fourteen of Peake's. Of the thirty-five, eighteen lie above and seventeen below 3°.182 C. cm.²/kgm. The mean of Grindley's values is 3.28_5, at 164°.5 C., or 3.27_0 at 165°, using $d\mu/dt = -0.031$ cm.²/kgm.; the mean of Griessmann's values is 3.10_0 at 162°.5, or 3.02_2 at 165°; the mean of Peake's values is 3.19_2 at 165°.5, or 3.20_8 at 165°. The mean of all thirty-five is $3°.17_9$ at 164°.2 or 3.15_4 at 165°. The actual vertical width of the band of points at 0.687 reduced temperature in Davis' figure 6 is about one-third of a reduced unit, or a little over 1° C. cm.²/kgm. This is about three times the difference between the two extreme values obtained by the present writer (See Table IV).

The precautions taken by these experimenters to avoid heat-leakage do not seem in all respects adequate, in view of the writer's experiences with this problem. Griessmann located his plug in a wooden pipe bound with iron hoops, and unlagged. The connected apparatus on both sides was well-lagged. Grindley lagged his low-side steam by means of a steam jacket supplied from the boiler used for the main steam supply. The temperature of the jacket steam could be regulated by means of throttle valves, but one infers from Grindley's paper that it was not always practicable to make it that of the low-side steam. Peake used a self-lagging arrangement on the low side of his plug. His steam passed from the plug into and through a glass tube, thence down on the outside of this tube. He also used thermally insulating joints for his low-side pressure and flow connections, but the writer's experience with similar joints is that they are of slight value. Only Peake appears to have made any systematic attempt to determine whether his results were in error from heat leakage. His plug consisted of a mica disc with a single small hole. Copper gauze was used on the low side to destroy the kinetic energy of the steam jet. Peake found that throttling curves obtained with identical high side conditions, but with orifices of different sizes, coincided in their overlapping portions, and from this inferred that heat-leakage was negligible. 'Coincidence' here must mean agreement within experimental error, which is probably not better than 0°.1 or 0°.2 C.

There is very little upon which to base a discussion of the probable effects of heat-leakage on the results of Grindley and Griessmann. Peake's affirmative tests for the absence of heat-leak effects can scarcely be regarded as of much authority in connection with the work of the other two experimenters, because of the difference between his lagging arrangements and either of theirs. It would seem quite

likely that Griessmann at least must have had a not inconsiderable outward heat-leak, the effect of which would have been to make his observed low-side temperatures lower than they would otherwise have been. Nevertheless, Griessmann's result for μ at 165° C. is the least of the three. It must be remembered in this connection that, because of the subtraction method employed in calculating μ from the data of all three observers, a considerable heat-leak may easily have been present without a proportionate effect, or even, necessarily, an effect of the same sign, on the calculated μ. But it is hardly conceivable that all of the discrepancy between the results of Griessmann and of Peake, or, still more, those of Griessmann and of Grindley, at 165° C., can be due to errors of observation; or, what amounts to the same thing, that the width of 1° C. cm.²/kgm. of the band of points of Davis' figure 6 can be due to this cause. It is not unlikely that the high value of Grindley's result is in part a low-side kinetic energy effect. His plug consisted of a quarter-inch glass plate through which a single $\frac{1}{16}$-inch hole was drilled, and his low side thermometer (a thermo-junction) was located in the jet from the orifice, about two inches from the orifice. He made tests to determine whether altering the position of the thermo-junction would affect the apparent low-side temperature, and concluded that, on the whole, the indication of the thermo-junction was independent of its position in the low-side channel. His tests do not seem absolutely convincing, however. Peake, who also used a single small hole (in a mica plate) as his main throttling device, found on one occasion, when he accidentally omitted the several layers of copper gauze regularly placed between the orifice and his low-side thermometer, that, under the most unfavorable conditions, the temperature registered by the low side thermometer was thereby depressed by 7° C. Peake's maximum flow was nearly twice Grindley's, however.

Aside from these probabilities, one must ascribe a part of the lack of agreement among Grindley, Griessmann and Peake to heat-leakage effects about which it is impossible to be in any degree specific. It happens that the result of that one of the three observers who made tests showing that his heat leakage effects were negligible is the most nearly in agreement with the value which the writer's work leads him to believe correct. For the rest, the most that can be said is that the effects of heat leakage were probably small and may have been of both signs, and thus minimized in a mean of the results.

Goodenough's empirical equation (equation (5)) gives, for μ at $p = 4.0$ kgm./cm.,² $T = 165 + 273$° C. abs., the value $\mu = 3°.34$ C.

cm.²/kgm. as compared with the value 3°.182 C. cm.²/kgm. obtained by the present writer.

V. SUMMARY.

1. In arriving at an experimental method of controlling and eliminating the effect of heat leakage in Joule-Thomson experiments, one type of axial flow, and three types of radial flow throttling apparatus, were used, and are described in detail. Broadly speaking, an axial flow throttling apparatus may be defined as one in which the direction of flow of the fluid is in general perpendicular to the leakage temperature gradient, and a radial flow throttling apparatus as one in which the plug, or throttling partition, is so shaped as to cause the fluid to flow through it in a direction generally parallel to the leakage temperature gradient.

Attention is called to the possibility of a secondary leakage effect (called the regeneration effect) in radial flow apparatus. The remedy for this effect is internal lagging.

2. Other apparatus, incidental to Joule-Thomson experiments and used in this research, is described. Particular attention is paid to the fundamental measurements of pressure and temperature differences, and the results of an experimental study of several important details connected with these measurements are given.

3. A short discussion of possible methods of eliminating heat-leakage effects from the immediate data of a set of throttling experiments is given.

4. The results of a large number of adiabatic Joule-Thomson experiments on superheated steam, with four axial flow plugs and five radial flow plugs, are presented and discussed in detail, chiefly as an experimental study of heat leakage. The best of the four particular axial flow plugs used is found to be incapable of dependable results without an excessive amount of experimental work. Evidence pointing to the probable presence of a small regeneration effect is shown to exist in the results obtained with certain radial flow plugs. Affirmative evidence of the absence of error from moisture in the steam is given, with a short discussion of experimental methods for avoiding this difficulty. It is shown that the results obtained with the final form of throttling apparatus — one of the radial flow type with heavy internal lagging — may reasonably be supposed to be free from appreciable heat-leak errors. Such results may also be

obtained without a great outlay of time in experimental work. While the radial flow plug thus appears to be much superior to the axial flow plug, it is pointed out that the axial flow plugs used in this research have a serious structural defect which a better design would avoid; hence the superiority of the radial flow plug, though doubtless actual, is not so great as a cursory inspection of the results exhibited graphically would indicate. The final apparatus is designed to permit more exact tests of the relative merits of the two types of plug: these have not as yet been made.

5. The value 3.182 degrees Centigrade per kilogram per square centimeter is given for the Joule-Thomson coefficient in steam at 165° C. and 3.86 kgm./cm.^{2} This is believed to be reliable to within 0.5 per cent.

6. The results of some early isothermal experiments giving directly the product μC_p are briefly summarized. An agreement within about 2.5 per cent. with the above value of μ is obtained by dividing the mean μC_p as derived from these experiments by Knoblauch and Mollier's value of C_p.

7. The question of the existence of a pressure coefficient of μ is discussed. This is of interest because the experiments from which the above value of μ is derived were not conducted at exactly the same mean pressure and were not corrected for the existing small differences in their mean pressures. A few experiments made by the writer indicate the existence of a small positive pressure coefficient. This is in verification of the recent steam equation of Goodenough. It is believed that the resulting correction in the above value of μ, which can not yet be made accurately, is less than 0.3 per cent.

8. It is shown that the above value of μ is in good agreement (within less than 1 per cent.) with the mean of the results of Grindley, Peake and Griessmann at the same temperature.

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